Vocabulary
Inductive reasoning is reasoning based on patterns you observe.

A conjecture is a conclusion you reach using inductive reasoning.

A counterexample is an example for which the conjecture is incorrect.

Examples
1. Finding and Using a Pattern: Find a pattern for the sequence: 34, 38, 42, ...
Each term is 4 more than the preceding term. The next two terms are 46 and 50.

2. Using Inductive Reasoning: Make a conjecture about the sum of the cubes of the first 25 counting numbers.
The sum of the first two cubes equals the square of the sum of the first two counting numbers. The sum of the first three cubes equals the square of the sum of the first three counting numbers. This pattern continues for the fourth and fifth rows. So a conjecture might be that the sum of the cubes of the first 25 counting numbers is $1^3 + 2^3 + 3^3 + \ldots + 25^3$.

Each year the price increased by $1.00. A possible conjecture is that the price in 2003 will increase by $1.00. If so, the price in 2003 would be $8.00 + 1.00 = $9.00.

Quick Check
1. Find the next two terms in each sequence.
   a. 1, 2, 3, 4, 5, 6, ..., ...
   b. Monday, Tuesday, Wednesday, ...

Answers may vary. Sample: a. 6, 7 b. Thursday, Friday

Local Standards: ____________________________________
Topic: Mathematical Reasoning
Geometry

Lesson 1-1 Patterns and Inductive Reasoning

Lesson Objectives
Vocabulary
Inductive reasoning is reasoning based on patterns you observe.

A conjecture is a conclusion you reach using inductive reasoning.

A counterexample is an example for which the conjecture is incorrect.

Examples
1. Finding and Using a Pattern: Find a pattern for the sequence: 34, 38, 42, ...
Each term is 4 more than the preceding term. The next two terms are 46 and 50.

2. Using Inductive Reasoning: Make a conjecture about the sum of the cubes of the first 25 counting numbers.
The sum of the first two cubes equals the square of the sum of the first two counting numbers. The sum of the first three cubes equals the square of the sum of the first three counting numbers. This pattern continues for the fourth and fifth rows. So a conjecture might be that the sum of the cubes of the first 25 counting numbers is $1^3 + 2^3 + 3^3 + \ldots + 25^3$.

Each year the price increased by $1.00. A possible conjecture is that the price in 2003 will increase by $1.00. If so, the price in 2003 would be $8.00 + 1.00 = $9.00.

Quick Check
1. Find the next two terms in each sequence.
   a. 1, 2, 3, 4, 5, 6, ..., ...
   b. Monday, Tuesday, Wednesday, ...

Answers may vary. Sample: a. 6, 7 b. Thursday, Friday

Local Standards: ____________________________________
Topic: Mathematical Reasoning
Geometry
Orthographic Drawing

Make an orthographic drawing of the isometric drawing at right.

Orthographic drawings flatten the depth of a figure. An orthographic drawing shows three views. Because no edge of the isometric drawing is hidden in the top, front, and right views, all lines are solid.

Front
Top
Right

Foundation Drawing

Make a foundation drawing for the isometric drawing.

To make a foundation drawing, use the top view of the orthographic drawing.

Because the top view is formed from squares, show squares in the foundation drawing.

Identify the square that represents the tallest part. Write the number 2 in the back square to indicate that the back section is cubes high.

Write the number 1 in each of the two front squares to indicate that each front section is cube high.

Identifying Solids from Nets

Is the pattern a net for a cube?

If no, name two letters that will be opposite faces.

The pattern is a net because you fold it to form a cube. Fold squares A and C up to form the back and front of the cube. Fold D up to form a side. Fold E over to form the top. Fold F down to form another side.

After the net is folded, the following pairs of letters are on opposite faces:

A and B in the back and front faces
B and C in the back and front faces
D and F to the right and left side faces

Drawing a Net

Draw a net for the figure with a square base and four isosceles triangle faces. Label the net with its dimensions.

Think of the sides of the square base as hinges, and “unfold” the figure at these edges to form a net. The base of each of the four isosceles triangle faces is a side of the square. Write in the known dimensions.

Quick Check

1. Make an isometric drawing of the cube structure below.

2. Make an isometric drawing from this isometric drawing.

3. a. How many cubes would you use to make the structure in Example 3?

b. Critical Thinking Which drawing did you use to answer part (a), the foundation drawing or the isometric drawing? Explain.

Answers may vary. Sample: the foundation drawing; you can just add the three numbers.

4. Sketch the three-dimensional figure that corresponds to the net.

5. The drawing shows one possible net for the Graham Crackers box. Draw a different net for this box. Show the dimensions in your diagram.

Answers may vary. Example:

GRAHAM CRACKERS

14 cm
7 cm
20 cm
14 cm
20 cm

14 cm
7 cm

Lesson 1-3 Points, Lines, and Planes

Vocabulary and Key Concepts.

Quick Check.

1. Use the figure in Example 1.

4. a. Name line and .
   b. Name a point that is coplanar with points , , and .
   c. Why do you think arrowheads are used when drawing a line?

Name three different names for plane .

Using Postulate 1-4

Collinear points are points that lie on the same line.

A line is a set of points that extends in two opposite directions without end.

A point is a location.

Space is the set of all points.

Collinear points are, any points that lie on the same line.

Postulate 1-1

Through any two points there is exactly one line.

Postulate 1-2

Through any two points there is exactly one plane.

Postulate 1-3

Any other set of three points in the figure do not lie on a line, so they are collinear.

Postulate 1-4

Through any three noncollinear points there is exactly one plane.

Postulate 1-5

If two planes interact, then they intersect in exactly one line.

Postulate 1-6

If two lines intersect, then they intersect in exactly one point.

Postulate 1-7

Line is the only line that passes through points and .

Examples

1. Identifying Collinear Points. In the figure at right, name three points that are collinear and three points that are not collinear.

   Points and lie on a line, so they are collinear.

   Any other set of three points in the figure do not lie on a line, so no other set of three points is collinear. For example, , , and .

   And are not collinear.

2. Naming a Plane. Name the plane shown in two different ways.

   The back and left faces of the cube intersect at . Planes and intersect vertically at .

3. Finding the Intersection of Two Planes. Use the diagram at right. What is the intersection of planes and ?

   As you look at the cube, the front face is on plane , the back face is on plane , and the left face is on plane .

   The back and left faces of the cube intersect at . Planes and intersect vertically at .

4. Using Postulate 1-4. Shade the planes that contain , and .

   Points , , and are the vertices of one of the four triangular faces of the pyramid. To shade the plane, shade the interior of the triangle formed by , , and .

   Some possible names for the plane shown are the following:

   Plane and plane ;
   Plane and plane ;
   Plane and plane .

Geometry: All-In-One Answers Version A (continued)

Lesson 1-5 Measuring Segments

1. Critical Thinking
   Name all segments that are parallel to \( \overleftrightarrow{EF} \). Name all segments that are skew to \( \overleftrightarrow{EF} \).

2. Using the figure in Example 1, and form a line. Are \( \overrightarrow{AE} \) and \( \overrightarrow{CB} \) congruent?

3. Use the diagram in Example 2.
   a. Name all labeled segments that are parallel to \( \overrightarrow{AB} \).
   b. Name all labeled segments that are skew to \( \overrightarrow{AB} \).
   c. Name three pairs of parallel segments and another pair of skew segments.

Answers may vary.

4. Use the diagram to the right.
   a. Name a line that is parallel to \( \overrightarrow{MN} \).
   b. Name a line that is parallel to plane \( P \).
   c. Name a line that is parallel to plane \( Q \).

Answers may vary.

Check Your Understanding
1. a. Name all labeled segments that are parallel to \( \overrightarrow{MN} \).
   b. Name all labeled segments that are skew to \( \overrightarrow{MN} \).
   c. Name three pairs of parallel segments and another pair of skew segments.

2. a. Use the diagram in Example 2.
   b. Name all labeled segments that are skew to \( \overrightarrow{EF} \).
   c. Name another pair of parallel segments and another pair of skew segments.

Answers may vary.

Examples

1. Naming Segments and Rays
   - Name the segments and rays in the figure.
   - b. Name the segments and rays in the figure.

2. Measuring Segments
   - a. Find the length of \( AB \).
   - b. Find the length of \( BC \).

3. Critical Thinking
   - a. Are \( \overrightarrow{AB} \) and \( \overrightarrow{CD} \) congruent?
   - b. Are \( \overrightarrow{AB} \) and \( \overrightarrow{CD} \) skew?

Answers may vary.

Lesson 5 Measuring Segments

Lesson Objectives
- Find the length of a segment
- Find the midpoint of a segment
- Use the Segment Addition Postulate
- Use the Distance Postulate
- Use the Perpendicular Postulate
- Use the Parallel Postulate
- Use the Midsegment Theorem
- Use the perpendicular bisector theorem
- Use the coordinate plane

Critical Thinking
- a. Are \( \overrightarrow{AB} \) and \( \overrightarrow{CD} \) congruent?
   - b. Are \( \overrightarrow{AB} \) and \( \overrightarrow{CD} \) skew?

Answers may vary.

Examples

1. Comparing Segment Lengths
   - Find \( AB \) and \( BC \).
   - Find \( AB \) and \( CD \).
   - Find \( AB \) and \( CD \).

2. Using the Segment Addition Postulate
   - Find \( AB \).
   - Find \( BC \).
   - Find \( CD \).

3. Using the Distance Postulate
   - Find \( AB \).
   - Find \( BC \).
   - Find \( CD \).

4. Using the Perpendicular Postulate
   - Find \( AB \).
   - Find \( BC \).
   - Find \( CD \).

5. Using the Midsegment Theorem
   - Find \( AB \).
   - Find \( BC \).
   - Find \( CD \).

6. Using the Perpendicular Bisector Theorem
   - Find \( AB \).
   - Find \( BC \).
   - Find \( CD \).

7. Using the Coordinate Plane
   - Find \( AB \).
   - Find \( BC \).
   - Find \( CD \).

Example

1. Find the length of \( AB \).
   - a. \( AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \)
   - b. \( AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \)
   - c. \( AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \)

2. Use the diagram in Example 2.
   - a. Name all labeled segments that are parallel to \( \overrightarrow{MN} \).
   - b. Name all labeled segments that are skew to \( \overrightarrow{MN} \).
   - c. Name another pair of parallel segments and another pair of skew segments.

Answers may vary.

Check Your Understanding
1. a. Name all labeled segments that are parallel to \( \overrightarrow{MN} \).
   - b. Name all labeled segments that are skew to \( \overrightarrow{MN} \).
   - c. Name three pairs of parallel segments and another pair of skew segments.

2. a. Use the diagram in Example 2.
   - b. Name all labeled segments that are skew to \( \overrightarrow{EF} \).
   - c. Name another pair of parallel segments and another pair of skew segments.

Answers may vary.

Examples

1. Comparing Segment Lengths
   - Find \( AB \) and \( BC \).
   - Find \( AB \) and \( CD \).
   - Find \( AB \) and \( CD \).

2. Using the Segment Addition Postulate
   - Find \( AB \).
   - Find \( BC \).
   - Find \( CD \).

3. Using the Distance Postulate
   - Find \( AB \).
   - Find \( BC \).
   - Find \( CD \).

4. Using the Perpendicular Postulate
   - Find \( AB \).
   - Find \( BC \).
   - Find \( CD \).

5. Using the Midsegment Theorem
   - Find \( AB \).
   - Find \( BC \).
   - Find \( CD \).

6. Using the Perpendicular Bisector Theorem
   - Find \( AB \).
   - Find \( BC \).
   - Find \( CD \).

7. Using the Coordinate Plane
   - Find \( AB \).
   - Find \( BC \).
   - Find \( CD \).

Example

1. Find the length of \( AB \).
   - a. \( AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \)
   - b. \( AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \)
   - c. \( AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \)

2. Use the diagram in Example 2.
   - a. Name all labeled segments that are parallel to \( \overrightarrow{MN} \).
   - b. Name all labeled segments that are skew to \( \overrightarrow{MN} \).
   - c. Name another pair of parallel segments and another pair of skew segments.

Answers may vary.

Check Your Understanding
1. a. Name all labeled segments that are parallel to \( \overrightarrow{MN} \).
   - b. Name all labeled segments that are skew to \( \overrightarrow{MN} \).
   - c. Name three pairs of parallel segments and another pair of skew segments.

2. a. Use the diagram in Example 2.
   - b. Name all labeled segments that are skew to \( \overrightarrow{EF} \).
   - c. Name another pair of parallel segments and another pair of skew segments.

Answers may vary.
Lesson 1-6 Measuring Angles

Postulate 1-5: Protractor Postulate
Let \(\overline{AB}\) and \(\overline{CD}\) be opposite rays in a plane. \(\overline{AB}\) and \(\overline{CD}\) can be paired with the real number from 0 to 180 so that:

a. \(\overline{AB}\) is paired with 80 and \(\overline{CD}\) is paired with 100.

b. If \(\overline{AB}\) is paired with \(x\) and \(\overline{CD}\) is paired with \(y\), then \(x + y = \text{measure of angle between} \overline{AB} \text{ and } \overline{CD}\).

Postulate 1-6: Angle Addition Postulate
If point \(B\) is in the interior of \(\angle AOC\), then
\[
\text{m} \angle AOB + \text{m} \angle BOC = \text{m} \angle AOC.
\]
If \(\angle AOC\) is a straight angle, then
\[
\text{m} \angle AOB + \text{m} \angle BOC = 180^\circ.
\]

Quick Check
1. Find \(x\) if \(m \angle AOB = 50^\circ\) and \(m \angle BOC = 130^\circ\).
2. Draw \(\overline{AB}\) and \(\overline{CD}\) so that \(\overline{AB}\) is a straight angle. Is \(\overline{CD}\) ever a straight line when \(\overline{AB}\) is a straight line?

Examples
1. Naming Angles: Name the angle at right in four ways.
2. Measuring and Classifying Angles: Find the measures of each \(\angle AOC\).
   - If the angles are acute, right, obtuse, or straight.
Lesson 1-7 Basic Constructions

**Vocabulary.**

1. **Examples.**

- **Constructing Congruent Segments:** Construct \( \overline{AB} \) congruent to \( \overline{CD} \).

  **Step 1:** Draw a ray with endpoint \( Y \).

  **Step 2:** Open the compass to the length of \( \overline{XZ} \).

  **Step 3:** With the same compass setting, put the compass point on point \( T \). Draw an arc that intersects the ray. Label the point of intersection \( W \).

  \( \overline{XZ} = \overline{CD} \)

- **Constructing Congruent Angles:** Construct \( \angle Y \) so that \( \angle Y \equiv \angle G \).

  **Step 1:** Draw a ray with endpoint \( Y \).

  **Step 2:** With the compass point on point \( G \), draw an arc that intersects both sides of \( \angle G \). Label the points of intersection \( E \) and \( F \).

  **Step 3:** With the same compass setting, put the compass point on point \( T \). Draw an arc that intersects the ray. Label the point of intersection \( Z \).

  **Step 4:** Open the compass to the length \( EF \). Keeping the same compass setting, put the compass point on \( Z \). Draw an arc that intersects with the arc you drew in Step 3. Label the point of intersection \( X \).

  **Step 5:** Draw \( \overline{XZ} \) to complete \( \angle Y \).

  \( \overline{Y} \equiv \angle G \)
Examples.

1. Given \( \overline{AB} \) with endpoints \( A (2, 3) \) and \( B (-1, 5) \).

   **a.** Find the midpoint of \( \overline{AB} \).
   
   **b.** Find the distance between points \( A \) and \( B \).

   **c.** Find the slope of \( \overline{AB} \).

2. Given \( \overline{XY} \) with endpoints \( X (3, 2) \) and \( Y (7, 6) \).

   **a.** Find the midpoint of \( \overline{XY} \).
   
   **b.** Find the distance between points \( X \) and \( Y \).

   **c.** Find the slope of \( \overline{XY} \).

3. Find the coordinates of the midpoint \( M \) of \( \overline{AB} \) where \( A (2, 3) \) and \( B (-1, 5) \).

   **a.** Use the Midpoint Formula.
   
   **b.** Use a calculator.

   Find the coordinates of the midpoint \( M \) of \( \overline{XY} \) where \( X (3, 2) \) and \( Y (7, 6) \).

   **a.** Use the Midpoint Formula.
   
   **b.** Use a calculator.

   The midpoint of \( \overline{AC} \) is \( (x_m, y_m) \) with endpoints \( A (2, 3) \) and \( C (5, 7) \).

   **a.** Use the Midpoint Formula.
   
   **b.** Use a calculator.

   The midpoint of \( \overline{DE} \) is \( (x_m, y_m) \) with endpoints \( D (0, 4) \) and \( E (6, 2) \).

   **a.** Use the Midpoint Formula.
   
   **b.** Use a calculator.

   The midpoint of \( \overline{FG} \) is \( (x_m, y_m) \) with endpoints \( F (1, -2) \) and \( G (3, 4) \).

   **a.** Use the Midpoint Formula.
   
   **b.** Use a calculator.

   The midpoint of \( \overline{HI} \) is \( (x_m, y_m) \) with endpoints \( H (-3, 5) \) and \( I (2, -1) \).

   **a.** Use the Midpoint Formula.
   
   **b.** Use a calculator.

   The midpoint of \( \overline{JK} \) is \( (x_m, y_m) \) with endpoints \( J (4, -2) \) and \( K (0, 3) \).

   **a.** Use the Midpoint Formula.
   
   **b.** Use a calculator.

   The midpoint of \( \overline{LM} \) is \( (x_m, y_m) \) with endpoints \( L (-1, 3) \) and \( M (5, 7) \).

   **a.** Use the Midpoint Formula.
   
   **b.** Use a calculator.

   The midpoint of \( \overline{MN} \) is \( (x_m, y_m) \) with endpoints \( M (2, 3) \) and \( N (-6, 7) \).

   **a.** Use the Midpoint Formula.
   
   **b.** Use a calculator.

   The midpoint of \( \overline{OP} \) is \( (x_m, y_m) \) with endpoints \( O (1, 2) \) and \( P (7, 6) \).

   **a.** Use the Midpoint Formula.
   
   **b.** Use a calculator.

   The midpoint of \( \overline{QR} \) is \( (x_m, y_m) \) with endpoints \( Q (3, 2) \) and \( R (7, 6) \).

   **a.** Use the Midpoint Formula.
   
   **b.** Use a calculator.

   The midpoint of \( \overline{ST} \) is \( (x_m, y_m) \) with endpoints \( S (0, 4) \) and \( T (6, 2) \).

   **a.** Use the Midpoint Formula.
   
   **b.** Use a calculator.

   The midpoint of \( \overline{UV} \) is \( (x_m, y_m) \) with endpoints \( U (1, -2) \) and \( V (3, 4) \).

   **a.** Use the Midpoint Formula.
   
   **b.** Use a calculator.

   The midpoint of \( \overline{WX} \) is \( (x_m, y_m) \) with endpoints \( W (4, -2) \) and \( X (0, 3) \).

   **a.** Use the Midpoint Formula.
   
   **b.** Use a calculator.

   The midpoint of \( \overline{YZ} \) is \( (x_m, y_m) \) with endpoints \( Y (2, -2) \) and \( Z (6, 7) \).

   **a.** Use the Midpoint Formula.
   
   **b.** Use a calculator.

   The midpoint of \( \overline{ABCD} \) is \( (x_m, y_m) \) with endpoints \( A (2, 3) \), \( B (5, 7) \), \( C (1, -2) \), and \( D (3, 4) \).

   **a.** Use the Midpoint Formula.
   
   **b.** Use a calculator.

   The midpoint of \( \overline{EFGH} \) is \( (x_m, y_m) \) with endpoints \( E (0, 4) \), \( F (6, 2) \), \( G (1, -2) \), and \( H (3, 4) \).

   **a.** Use the Midpoint Formula.
   
   **b.** Use a calculator.

   The midpoint of \( \overline{IJLM} \) is \( (x_m, y_m) \) with endpoints \( I (4, -2) \), \( J (2, 3) \), \( K (7, 6) \), and \( L (2, 3) \).

   **a.** Use the Midpoint Formula.
   
   **b.** Use a calculator.

   The midpoint of \( \overline{OPQR} \) is \( (x_m, y_m) \) with endpoints \( O (1, 2) \), \( P (7, 6) \), \( Q (3, 2) \), and \( R (7, 6) \).

   **a.** Use the Midpoint Formula.
   
   **b.** Use a calculator.

   The midpoint of \( \overline{STUV} \) is \( (x_m, y_m) \) with endpoints \( S (0, 4) \), \( T (6, 2) \), \( U (1, -2) \), and \( V (3, 4) \).

   **a.** Use the Midpoint Formula.
   
   **b.** Use a calculator.

   The midpoint of \( \overline{WXYZ} \) is \( (x_m, y_m) \) with endpoints \( W (4, -2) \), \( X (0, 3) \), \( Y (2, -2) \), and \( Z (6, 7) \).

   **a.** Use the Midpoint Formula.
   
   **b.** Use a calculator.
3.**Quick Check.**

1. **a.** Finding the midpoint of \(XY\) with endpoints \((1, 4)\) and \((6, 9)\).

2. **b.** Find the coordinates of the midpoint of \(XY\) with endpoints \((2, 1)\) and \((4, 3)\).

3. The midpoint of \(XY\) has coordinates \((6, 0)\). Find the coordinates of \(Y\) given \(X\) has coordinates \((2, 3)\).

4. **b.** Find the coordinates of the midpoint of \(MN\) with endpoints \((-1, 2)\) and \((4, 6)\).
Lesson 2-1 Conditional Statements

Vocabulary and Key Concepts
Conditional Statements and Converses
Example
Write the converse of the following conditional:

1. If two lines are not parallel and do not intersect, then they are skew.
   Converse: The converse exchanges the hypothesis and the conclusion.

2. If two lines do not intersect, then they are parallel.
   Converse: The converse exchanges the hypothesis and the conclusion.

Finding the Truth Value of a Converse
In a conditional statement, the clause after the hypothesis is the hypothesis and the clause after the conclusion is the conclusion.

Identifying the Hypothesis and the Conclusion
Identifying the hypothesis and conclusion:
- Hypothesis: If two lines are parallel, then the lines are coplanar.
- Conclusion: The conditional is true and the converse is false.

The truth value of a statement is true or false, respectively.

Recognize conditional statements
Statement Example Symbolic Form You read it

Conditional Statements and Converses

Local Standards: ____________________________________

Topic: Dimension and Shape; Mathematical Reasoning

NAEP 2005 Strand: Geometry

Lesson 2-2 Biconditionals and Definitions

Vocabulary and Key Concepts

Biconditional Statements
A biconditional combines $p \rightarrow q$ and $q \rightarrow p$.

Statement
Example
Symbolic Form
You read it

Biconditional
An angle is a straight angle if and only if its measure is 180°.
$p \rightarrow q$ and
$q \rightarrow p$

Biconditional: A biconditional contains the words "if and only if." A biconditional combines the conditional and its converse.

Examples

1. Writing a Biconditional
   Consider the true conditional statement: Write its converse:
   
   If the name of a state includes the word "New," then it does not border an ocean.
   
   Converse: New Mexico does not border an ocean.
   
Quick Check
1. Identify the hypothesis and conclusion of the following conditional statement:
   If $x = 3$, then $y = 8$.
   
   Hypothesis: $x = 3$.
   
   Conclusion: $y = 8$.
   
   2. Show that this conditional is false by finding a counterexample:
   If the name of a state includes the word "New," then it does not border an ocean.
   
   Counterexample:
   New Mexico does not border an ocean, it is a counterexample, so the conditional is false.

Local Standards: ____________________________________

Topic: Dimension and Shape; Mathematical Reasoning

NAEP 2005 Strand: Geometry

Lesson Objective:
- Write biconditionals.
- Recognize definitions.

Local Standards:
- Dimension and Shape; Mathematical Reasoning
- Geometry
Lesson 2-3

Deductive Reasoning

Lesson Objectives
- Use the Law of Detachment
- Use the Law of Syllogism

vocabulary and Key Concepts

Law of Detachment
If a conditional is true and its hypothesis is true, then its conclusion is true.

In symbolic form: $p \to q$, $p$, then $q$.

Law of Syllogism
If $p \to q$ and $q \to r$ are true statements, then $p \to r$ is a true statement.

Deductive reasoning is a process of reasoning logically from given facts to a conclusion.

Examples

1. **Using the Law of Detachment:** A gardener knows that if it rains, the garden will be watered. It is raining. What can you conclude?

   - **Conclusion:** The garden will be watered.

2. **Using the Law of Detachment:** The statement is true as a description of an apple.

   - **Use the Law of Detachment:**

Quick Check

1. Consider the true conditional statement. Write its converse.

   - **Statement:** If three points are collinear, then they lie on the same line.
   - **Converse:** Three points are collinear if and only if they lie on the same line.

2. Write two statements that form a true biconditional about integers greater than 1:

   - **Statement:** A number is prime if and only if it has only two distinct factors, 1 and itself.
   - **Converse:** A number has only two distinct factors, 1 and itself, then it is prime.

3. Show that this definition of right angle is reversible. Then write it as a true biconditional.

   - **Definition:** A right angle has measure 90°.
   - **Converse:** If an angle has measure 90°, then it is a right angle.

4. Is the following statement a good definition? Explain.

   - **Statement:** An angle is a right angle if and only if its measure is 90°.
   - **Conclusion:** It is not a good definition because a rectangle has four right angles and is not necessarily a square.

Using the Law of Syllogism

- Use the Law of Syllogism to draw a conclusion from the following true statements:

   - **Law of Syllogism:**
     - **Premise:** If $p$, then $q$.
     - **Premise:** If $q$, then $r$.
     - **Conclusion:** If $p$, then $r$.

Drawing Conclusions

- Use the Laws of Detachment and Syllogism to draw a possible conclusion.

   - **Premise:** If the circus is in town, then there are tents at the fairground. If there are tents at the fairground, then Paul is working as a night watchman.
   - **Conclusion:** If the circus is in town, then Paul is working as a night watchman.

Quick Check

1. Suppose that a mechanic begins work on a car and finds that the car will not start. Can the mechanic conclude that the car has a dead battery? Explain.

   - **Conclusion:** No, there could be other things wrong with the car, such as a faulty starter.

2. If a baseball player is a pitcher, then that player should not pitch complete games two days in a row. Vladimir Nuñez is a pitcher. On Monday, he pitches a complete game. What can you conclude?

   - **Answer:** Sample: Vladimir Nuñez should not pitch a complete game on Tuesday.
Use the Law of Detachment and the Law of Syllogism to draw conclusions.

4. If possible, state a conclusion using the Law of Syllogism. If it is not possible to use this law, explain why.
   - If a number ends in 6, then it is divisible by 2.
   - If a number ends in 0, then it is divisible by 10.
   - If a number ends in 0, then it is divisible by 5.

   a. Conclusion:

   b. Conclusion:

   The Volga River is in Europe.
   - The Volga River is not one of the world’s ten longest rivers.

   Conclusion:
   - The Volga River is not one of the world’s ten longest rivers.
Lesson 2-5

Proving Angles Congruent

Vocabulary and Key Concepts

Theorem 2-1: Vertical Angles Theorem
Vertical angles are congruent.

Theorem 2-2: Congruent Supplements Theorem
If two angles are supplements of the same angle (or of congruent angles), then the two angles are congruent.

Theorem 2-3: Congruent Complements Theorem
If two angles are complements of the same angle (or of congruent angles), then the two angles are congruent.

Examples

Using the Vertical Angles Theorem. Find the value of x.

\[ m\angle 1 + m\angle 2 = 180 \text{°} \]

Start with the given:

\[ m\angle 1 + m\angle 2 = 180 \text{°} \]

Using the Subtraction Property of Equality:

\[ m\angle 1 = m\angle 2 = 90 \text{°} \]

Proving Theorem 2-2: Write a paragraph proof of Theorem 2-2 using the diagram at the right.

Given:

\[ \angle 1 \text{ and } \angle 2 \text{ are supplementary} \]

Prove:

\[ \angle 1 \text{ and } \angle 2 \text{ are congruent} \]

Statements

1. \[ \angle 1 \text{ and } \angle 2 \text{ are supplementary} \]
2. \[ m\angle 1 + m\angle 2 = 180 \text{°} \]
3. \[ m\angle 1 = m\angle 2 \]
4. \[ \angle 1 \text{ and } \angle 2 \text{ are congruent} \]

Reasons

1. Given
2. Vertical Angles Theorem
3. \[ m\angle 1 = m\angle 2 \]
4. Definition of Congruent Angles

Lesson 3-1

Properties of Parallel Lines

Vocabulary and Key Concepts

Postulates 3-1: Corresponding Angles Postulate
If a transversal intersects two parallel lines, then corresponding angles are congruent.

Theorem 3-1: Alternate Interior Angles Theorem
If a transversal intersects two parallel lines, then alternate interior angles are congruent.

Theorem 3-2: Same-Side Interior Angles Theorem
If a transversal intersects two parallel lines, then same-side interior angles are supplementary.

Theorem 3-3: Alternate Exterior Angles Theorem
If a transversal intersects two parallel lines, then alternate exterior angles are congruent.

Theorem 3-4: Same-Side Exterior Angles Theorem
If a transversal intersects two parallel lines, then same-side exterior angles are supplementary.
Examples

**Applying Properties of Parallel Lines**

In the diagram at right, \( \overrightarrow{ED} \) and \( \overrightarrow{EF} \) are parallel, and \( \overrightarrow{EG} \) is a transversal.

1. Classify the angles:
   - \( \angle 1 \) and \( \angle 2 \) are corresponding angles.
   - \( \angle 3 \) and \( \angle 4 \) are alternate interior angles.
   - \( \angle 5 \) and \( \angle 6 \) are same-side interior angles.

2. Find the values of \( a \), \( b \), and \( c \):
   - \( a = 38 \)°
   - \( b = 152 \)°
   - \( c = 112 \)°

**Using Algebra to Find Angle Measures**

To find the value of \( x \):
- \( 2x + 50 = 150 \)
- \( 2x = 100 \)
- \( x = 50 \)°

**Using Algebra**

Find the value of \( c \) for which \( \angle 3 \) and \( \angle 4 \) are supplementary.

- \( \angle 3 \) and \( \angle 4 \) are supplementary:
- \( m\angle 3 + m\angle 4 = 180 \)°
- \( c \) is the measure of \( \angle 4 \):
- \( c + (c + 10) = 180 \)
- \( 2c + 10 = 180 \)
- \( 2c = 170 \)
- \( c = 85 \)°

**Proving Lines Parallel**

A two-column proof is a display that shows the steps and reasons that prove statements.

1. \( \angle 2 \) and \( \angle 5 \) are corresponding angles.
2. \( \angle 3 \) and \( \angle 4 \) are alternate interior angles.
3. \( \angle 1 \) and \( \angle 6 \) are same-side interior angles.

**Alternate Interior Angles**

If two lines and a transversal form alternate interior angles that are congruent, then the two lines are parallel.

**Corresponding Angles**

If two lines and a transversal form corresponding angles that are congruent, then the two lines are parallel.

**Same-Side Interior Angles**

If two lines and a transversal form same-side interior angles that are supplementary, then the two lines are parallel.
Quick Check.

1. Use the diagram from Example 1. Which lines, if any, must be parallel if \( m \angle 1 = m \angle 2 \)? Explain.

2. From what is given in Example 2, can you also conclude that the transversal is perpendicular to \( m \angle 3 \)? Explain.

Lesson 3-3 Parallel and Perpendicular Lines

**Lesson Objectives**
- Identify parallel and perpendicular lines
- Use the Properties of Parallel Lines

**Local Standards:**

**Vocabulary and Key Concepts.**

**Examples.**

**Real-World Connection**

A picture frame is assembled as shown. Given the book’s explanation for why the outer edges on opposite sides of the frame are parallel, why must the inner edges on opposite sides be parallel, too?

**Keywords:**
- parallel lines
- perpendicular lines
- corresponding angles
- alternate interior angles
- consecutive interior angles

**Key Concepts.**

**Theorems:**

**Theorem 3-9**

If two lines are parallel, then they are to each other.

**Theorem 3-10**

In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.

**Theorem 3-11**

In a plane, if two lines are perpendicular to one of two parallel lines, then they are parallel to each other.

**Examples.**

1. A picture frame is assembled as shown. Given the book’s explanation for why the outer edges on opposite sides of the frame are parallel, why must the inner edges on opposite sides be parallel, too?

2. Use the diagram from Example 1. Which lines, if any, must be parallel if \( m \angle 1 = m \angle 2 \)? Explain.

**Skills Practice.**

1. Use the diagram from Example 1. Which lines, if any, must be parallel if \( m \angle 1 = m \angle 2 \)? Explain.

2. From what is given in Example 2, can you also conclude that the transversal is perpendicular to line \( \ell \)? Explain.
Lesson 3-5 The Polygon Angle-Sum Theorems

Vocabulary and Key Concepts:

- **Theorem 3-14: Polygon Angle-Sum Theorem**
  - The sum of the measures of the angles of an $n$-gon is $180(n-2)$.

- **Theorem 3-15: Polygon Exterior Angle-Sum Theorem**
  - The sum of the measures of the exterior angles of a polygon is $360^\circ$.

- **Polygon**
  - A closed plane figure with at least three sides that are segments. The sides intersect only at their endpoints, and no two adjacent sides are collinear.

- **Convex Polygon**
  - A polygon in which all interior angles are less than $180^\circ$.

- **Concave Polygon**
  - A polygon in which at least one interior angle is greater than $180^\circ$.

- **Diagonal**
  - A segment connecting two nonadjacent vertices of a polygon.

Examples:

1. **Classifying Polygons**
   - Classify the polygons by their sides. Identify if they are convex or concave.

   - **Example 1:**
     - Given a quadrilateral with sides $a$, $b$, $c$, and $d$, classify the polygon as convex or concave.
     - **Solution:**
       - If $a+b+c+d < 360^\circ$, the polygon is convex.
       - If $a+b+c+d > 360^\circ$, the polygon is concave.

2. **Finding an Interior Angle of a Triangle**
   - Find the measure of each interior angle of a triangle.

   - **Example 2:**
     - Given a triangle with angles $A$, $B$, and $C$, find $m\angle A$.
     - **Solution:**
       - $m\angle A + m\angle B + m\angle C = 180^\circ$.
       - $A = 180^\circ - (B + C)$.

Quick Check:

1. Find $m\angle ABC$.
   - **Solution:**
     - $m\angle ABC = 180^\circ - (m\angle A + m\angle C)$.

2. Find $m\angle DEF$.
   - **Solution:**
     - $m\angle DEF = 180^\circ - (m\angle D + m\angle F)$.

Using the Polygon Angle-Sum Theorem:

- **Example 3:**
  - Find $m\angle X$ in quadrilateral $WXYZ$.
  - **Solution:**
    - $m\angle X + m\angle Y + m\angle W + m\angle Z = 360^\circ$.
    - $m\angle X = 360^\circ - (m\angle Y + m\angle W + m\angle Z)$.
Quick Check.

1. Find the sum of the measures of the angles of a 13-gon.

2. Critical Thinking
   - The sum of the measures of the angles of a given polygon is 1980°. The hexagon is regular, so all its angles are congruent. An exterior angle is the supplement of a polygon's angle because they are adjacent angles that form a straight angle. Because the measures of adjacent angles are congruent, all the angles marked 1 have equal measures.

Vocabulary.
- Vocabulary
  - A definition
  - Examples

Lesson Objectives
- Writing an Equation of a Line Given Two Points
- Using Point-Slope Form

Using Point-Slope Form
- Write an equation in point-slope form of the line with slope -3 that contains (2, -6).

Quick Check
- Graph each equation.
  - a. y = -3x - 2
  - b. 2x + 4y = -8
  - c. 5x + y = -3

Line Equations
- The slope-intercept form of a linear equation is \( y = mx + b \).

Writing an Equation of a Line Given Two Points
- Write an equation in point-slope form of the line that contains the points (4, -9) and (1, 1).

Quick Check
- 2. Write an equation of the line with slope -1 that contains the point (1, -4).

Local Standards: ____________________________________
- Patterns, Relations, and Functions;
- Algebraic Representations

All-In-One Answers Version A (continued)
Lesson 3-7
Slopes of Parallel and Perpendicular Lines

Name______________________ Class________________________ Date________________

Example 3

1. The equation $y = -5x + 4$ has a slope of $m = -5$. To find the slope of a line perpendicular to this line, we can use the concept that the slopes of perpendicular lines are negative reciprocals of each other.

$$m_{	ext{perpendicular}} = -rac{1}{m} = -rac{1}{-5} = rac{1}{5}$$

2. The line $x = -5y + 4$ can be written in slope-intercept form as $y = rac{1}{5}x - rac{4}{5}$. The slope of this line is $m = rac{1}{5}$, which is the same as the slope of the line perpendicular to $y = -5x + 4$. Therefore, the two lines are not parallel because their slopes are not equal.

Quick Check

1. Are the lines parallel? Explain. Yes; the lines have the same slope and y-intercept, so they are the same line.

2. Write an equation for the line parallel to $y = -x + 4$ that contains $(2, 5)$.

$$y - 5 = -1(x - 2)$$

$$y = -x + 7$$

3. Write an equation for the line perpendicular to $y = -x + 4$ that contains $(2, 5)$.

$$y - 5 = 1(x - 2)$$

$$y = x + 3$$

Local Standards: ____________________________________

Measuring Physical Attributes

NAEP 2005 Strand: Measurement

Slopes of Parallel Lines
If two nonvertical lines are parallel, their slopes are equal. Any two vertical lines are parallel.

Slopes of Perpendicular Lines
If two nonvertical lines are perpendicular, the product of their slopes is $-1$. Any horizontal line and vertical line are perpendicular.

Step 1

Determine Whether Lines Are Parallel

1. Are the lines $y = -5x + 4$ and $x = -5y + 4$ parallel? Explain.

The equation $y = -5x + 4$ is in slope-intercept form. Write the equation $x = -5y + 4$ in slope-intercept form:

$$x = -5y + 4$$

$$y = -rac{1}{5}x + rac{4}{5}$$

The slopes are not equal, so the lines are not parallel.

Quick Check

1. Are the lines parallel? Explain.

2. Write an equation for the line parallel to $y = -x + 4$ that contains $(2, 5)$.

3. Write an equation for the line perpendicular to $y = -x + 4$ that contains $(2, 5)$.

Local Standards: ____________________________________

Geometry: All-In-One Answers Version A (continued)

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### Lesson 4-1 Congruent Figures

**Lesson Objective**
- Describes congruent figures and their corresponding parts.

#### Vocabulary and Key Concepts

**Theorem 4-1**
Congruent polygons are polygons that have corresponding sides congruent and corresponding angles congruent.

**Naming Congruent Parts**
- \( \triangle ABC \cong \triangle DEF \)
- List the congruent corresponding parts.

**Using Congruency**
- Use the Triangle Angle-Side Theorem and the definition of congruent polygons to find \( m\angle X \).

- **Example**
  - Constructing a Special Quadrilateral: Construct a quadrilateral with both pairs of opposite sides parallel.
  - Step 1: Draw point \( A \) and two rays with endpoints at \( A \). Label point \( B \) on one ray and point \( C \) on the other ray.
  - Step 2: Construct a ray parallel to \( \overleftrightarrow{AB} \) through point \( B \).
  - Step 3: Construct a ray parallel to \( \overleftrightarrow{AC} \) through point \( C \).
  - Step 4: Label point \( D \) where the ray parallel to \( \overleftrightarrow{AC} \) intersects the ray parallel to \( \overleftrightarrow{AB} \).

**Quick Check**
1. Draw two segments. Label their lengths \( c \) and \( d \). Construct a quadrilateral with one pair of parallel sides of lengths \( c \) and \( d \).
2. a. Find \( \angle Y \).
   - \( \angle Y = 67 \) and \( m\angle M = 48 \). Find \( m\angle X \).
   - **Solution**
     - \( \angle Y = 67 \) and \( m\angle M = 48 \). Use the Triangle Angle-Side Theorem and the definition of congruent polygons to find \( m\angle X \).
     - \( m\angle X = m\angle Y + m\angle Z \)
     - \( m\angle Z = 115 + 180 \)
     - \( m\angle X = 395 \)

3. a. Find \( \angle Y \) if \( m\angle Z = 35 \), what is \( m\angle X \)?
   - **Solution**
     - \( m\angle X = m\angle Y + m\angle Z \)
     - \( m\angle Z = 35 \)
     - \( m\angle X = 35 + 115 \)
     - \( m\angle X = 150 \)

**Finding Congruent Triangles**
Can you conclude that \( \triangle ABC \cong \triangle DEF \)?
- List corresponding vertices in the same order.

- **Example**
  - Perpendicular From a Point to a Line: Construct a perpendicular from point \( F \) to line \( \overleftrightarrow{AC} \).
  - Point \( G \) is the same distance from point \( E \) as point \( A \) from point \( B \) because both arcs were made with the same compass opening.

**Quick Check**
1. a. Draw a straightedge to draw \( \overleftrightarrow{AC} \).
   - Construct \( \overleftrightarrow{AE} \) so that \( \overleftrightarrow{AC} \perp \overleftrightarrow{AE} \).
   - **Solution**
     - **Example**
     - Using Theorem 4-1, you can conclude that \( \overleftrightarrow{AC} \perp \overleftrightarrow{AE} \).

2. b. Draw a line \( \overleftrightarrow{AC} \) and a point \( F \) not on \( \overleftrightarrow{AC} \).
   - Construct \( \overleftrightarrow{AE} \) so that \( \overleftrightarrow{AC} \perp \overleftrightarrow{AE} \).

**Topic:**
- Transformation of Shapes and Preservation of Properties

**Local Standards:**
- __________________________________________

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Geometry: All-In-One Answers Version A (continued)

Lesson 4-2

Triangle Congruence by SSS and SAS

Key Concepts

Postulate 4-1: Side-Side-Side (SSS) Postulate
If the three sides of one triangle are congruent to the three sides of another triangle, then the two triangles are congruent.

Postulate 4-2: Side-Angle-Side (SAS) Postulate
If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.

Examples

Proving Triangles Congruent
Given: M is the midpoint of \( \overline{EF} \) and \( \overline{GH} \) is parallel to \( \overline{AB} \)
Prove: \( \triangle AEB \cong \triangle CGB \)

Write a paragraph proof.

You are given that \( M \) is the midpoint of \( \overline{EF} \). \( \overline{GH} \) is parallel to \( \overline{AB} \) by the

Reflexive Property of Congruence

\( \triangle AEB \cong \triangle AMX \) by the SAS Postulate.

Quick Check

1. Given: \( \overline{EF} \parallel \overline{GH} \)

2. \( M \) is the midpoint of \( \overline{EF} \)

3. \( \overline{GH} \parallel \overline{AB} \)

4. \( \triangle AEB \cong \triangle AMX \)

Statements

Reasons

1. \( \overline{EF} \parallel \overline{GH} \)

2. \( M \) is the midpoint of \( \overline{EF} \)

3. \( \overline{GH} \parallel \overline{AB} \)

4. \( \triangle AEB \cong \triangle AMX \)

Using SAS \( \overline{EF} \cong \overline{GH} \). What other information do you need to prove \( \triangle AEB \cong \triangle AMX \) by SAS?

It is given that \( \overline{EF} \parallel \overline{GH} \). Also, \( \overline{GH} \parallel \overline{AB} \) by the

Reflexive Property of Congruence

Two pairs of corresponding sides and their included angles are congruent, so \( \triangle AEB \cong \triangle AMX \) by the SAS Postulate.

Quick Check

1. Given: \( \overline{EF} \parallel \overline{GH} \)

2. \( M \) is the midpoint of \( \overline{EF} \)

3. \( \overline{GH} \parallel \overline{AB} \)

4. \( \triangle AEB \cong \triangle AMX \)

Statements

Reasons

1. \( \overline{EF} \parallel \overline{GH} \)

2. \( M \) is the midpoint of \( \overline{EF} \)

3. \( \overline{GH} \parallel \overline{AB} \)

4. \( \triangle AEB \cong \triangle AMX \)

Using SAS \( \overline{EF} \cong \overline{GH} \). What other information do you need to prove \( \triangle AEB \cong \triangle AMX \) by SAS?

It is given that \( \overline{EF} \parallel \overline{GH} \). Also, \( \overline{GH} \parallel \overline{AB} \) by the

Reflexive Property of Congruence

Two pairs of corresponding sides and their included angles are congruent, so \( \triangle AEB \cong \triangle AMX \) by the SAS Postulate.

Quick Check

1. Given: \( \overline{EF} \parallel \overline{GH} \)

2. \( M \) is the midpoint of \( \overline{EF} \)

3. \( \overline{GH} \parallel \overline{AB} \)

4. \( \triangle AEB \cong \triangle AMX \)

Statements

Reasons

1. \( \overline{EF} \parallel \overline{GH} \)

2. \( M \) is the midpoint of \( \overline{EF} \)

3. \( \overline{GH} \parallel \overline{AB} \)

4. \( \triangle AEB \cong \triangle AMX \)
Lesson 4-4 Using Congruent Triangles: CPCTC

**Lesson Objective**
- Use triangle congruence and CPCTC to prove that pairs of two triangles are congruent.

**Vocabulary**
- CPCTC (Corresponding Parts of Congruent Triangles)
- Corresponding
- Congruent

**Examples**
1. **Writing a Proof Using AAS**
   - **Given:** \( \triangle ABC \cong \triangle DEF \)
   - **Prove:** \( \angle A \cong \angle D \)
   - **Solution:**
     - By the Reflexive Property of Congruence, \( \angle A \cong \angle A \)
     - By the Alternate Interior Angles Theorem, \( \angle BAC \cong \angle EDF \)
     - Therefore, \( \triangle ABC \cong \triangle DEF \)
     - By CPCTC, \( \angle A \cong \angle D \)

2. **Planning a Proof**
   - **Given:** \( \triangle ABC \cong \triangle DEF \)
   - **Prove:** \( \angle B \cong \angle D \)
   - **Solution:**
     - By the Reflexive Property of Congruence, \( \angle B \cong \angle B \)
     - By the Alternate Interior Angles Theorem, \( \angle BAC \cong \angle EDF \)
     - Therefore, \( \angle B \cong \angle D \)

3. **Writing a Two-column proof that uses AAS**
   - **Given:** \( \triangle ABC \cong \triangle DEF \)
   - **Prove:** \( \angle A \cong \angle D \)
   - **Proof:**
     - By the Reflexive Property of Congruence, \( \angle A \cong \angle A \)
     - By the Alternate Interior Angles Theorem, \( \angle BAC \cong \angle EDF \)
     - Therefore, \( \angle A \cong \angle D \)

**Quick Check**
1. **Write a proof.**
   - **Given:** \( \triangle ABC \cong \triangle DEF \)
   - **Prove:** \( \angle A \cong \angle D \)
   - **Solution:**
     - By the Reflexive Property of Congruence, \( \angle A \cong \angle A \)
     - By the Alternate Interior Angles Theorem, \( \angle BAC \cong \angle EDF \)
     - Therefore, \( \angle A \cong \angle D \)
2. **Recall Example 2.**
   - **Given:** \( \triangle ABC \cong \triangle DEF \)
   - **Prove:** \( \angle A \cong \angle D \)
   - **Solution:**
     - By the Reflexive Property of Congruence, \( \angle A \cong \angle A \)
     - By the Alternate Interior Angles Theorem, \( \angle BAC \cong \angle EDF \)
     - Therefore, \( \angle A \cong \angle D \)
Lesson 4-5  Isosceles and Equilateral Triangles

Vocabulary and Key Concepts.

- **Theorem 4-3:** Isosceles Triangle Theorem
  - If a triangle is equilateral, then the triangle is equiangular.
  - The bisector of the vertex angle of an isosceles triangle is the perpendicular bisector of the base.

- **Corollary to Theorem 4-3:**
  - The measure of the angle marked by the bisector of the vertex angle of an isosceles triangle is the sum of the measures of the two angles that are not marked.

- **Theorem 4-4:**
  - The triangle is equilateral.

- **Corollary to Theorem 4-4:**
  - The measure of the angle marked by the bisector of the vertex angle of an isosceles triangle is the sum of the measures of the two angles that are not marked.

- **Example:**
  - Using the Isosceles Triangle Theorems

    - The diagram shows that \( \angle XAB = \angle ABC \) by the Isosceles Triangle Theorem.

- **Quick Check:**
  - \( m \angle L = y \) and \( m \angle N = y \)

- **Using Isosceles Triangles**
  - The garden shown at the right is in the shape of a regular hexagon. Suppose that a segment is drawn between the endpoints of the angle marked \( x \). Find the angle measures of the triangle that is formed.

- **Examples:**
  - Find the values of \( x \), \( y \), and \( z \).

- **Quick Check:**
  - Suppose \( m \angle L = 45 \). Find the values of \( x \) and \( y \).

- **Lesson 4-6  Congruence in Right Triangles

Vocabulary and Key Concepts.

- **Theorem 4-6:** Hypotenuse-Leg (HL) Theorem
  - If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and leg of another right triangle, then the triangles are congruent.

- **Examples:**
  - Proving Triangles Congruent

    - The diagram shows the following congruent parts:
      - \( \angle CPA = \angle MPB \) by the HL Theorem.

    - The student is correct. Explain.

    - The student correct. Explain.
Quick Check.

1. The diagram shows triangles from the scaffolding that workers used to build the 17005 scaffold. Name the common side in triangles XYZ and YXZ. One side is \( \overline{XY} \) and another is \( \overline{XZ} \). Identify the common parts of the sides that share these common sides. Write a plan for a proof that triangle XYZ is congruent to XYZ. Use the Reflexive Property of Congruence if necessary.

2. Write a paragraph proof of Example 2. It is given that \( \overline{XY} \) is perpendicular to \( \overline{FZ} \). This means that \( \angle YXF \) and \( \angle ZXF \) are right angles, since the definition of perpendicular lines is satisfied. Therefore, the sum of the measures of \( \angle YXF \) and \( \angle ZXF \) is 90 degrees. By the Reflexive Property of Congruence, \( \overline{XY} \) \( \cong \) \( \overline{XY} \). Since right angles are congruent, \( \angle YXF \) \( \cong \) \( \angle ZXF \) by the definition of isosceles triangles. YXM \( \cong \) \( \angle ZMX \) by the Reflexive Property of Congruence. Therefore, since \( \angle YXF \) \( \cong \) \( \angle ZXF \) and \( \angle YXF \) \( \cong \) \( \angle ZXF \), \( \angle YXM \) \( \cong \) \( \angle ZMX \) by CPCTC.

3. You know that two legs of one right triangle are congruent to two legs of another right triangle. Explain how to prove the triangles are congruent. Since all right angles are congruent, the triangles are congruent by SAS.
Write a two-column proof.

Given: \( \triangle ABC \), \( D \), and \( E \) are midpoints of \( AB \) and \( AC \), respectively.

Prove: \( DE = \frac{1}{2} \text{BC} \)

**Statement** | **Reason**
--- | ---
1. \( D \) is the midpoint of \( AB \) | **Given**
2. \( E \) is the midpoint of \( AC \) | **Given**
3. \( AD = DB \) | **Definition of midpoint**
4. \( AE = EC \) | **Definition of midpoint**
5. \( \triangle ADE \) and \( \triangle CDE \) are congruent | **Triangle Midsegment Theorem**
6. \( DE = \frac{1}{2} \text{BC} \) | **Definition of midpoint**

**Proof:**

1. \( D \) and \( E \) are midpoints of \( AB \) and \( AC \), respectively.
2. \( AD = DB \) and \( AE = EC \) by the definition of midpoint.
3. Therefore, \( \triangle ADE \) and \( \triangle CDE \) are congruent by the Triangle Midsegment Theorem.
4. Since \( \triangle ADE \) and \( \triangle CDE \) are congruent, \( DE = \frac{1}{2} \text{BC} \) by the definition of midpoint.

**Conclusion:** \( DE = \frac{1}{2} \text{BC} \).
Quick Check.

Vocabulary and Key Concepts.

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Use the information given in the diagram. is the perpendicular bisector of .

According to the diagram, how far is ? From ?

2. a. Find .

b. What can you conclude about ?

c. Find the value of .

The distance from a point to a line is the length of the perpendicular segment from the point to the line.

Theorem 5-2: Perpendicular Bisector Theorem

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

Theorem 5-3: Converse of the Perpendicular Bisector Theorem

If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

Theorem 5-4: Angle Bisector Theorem

If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.

Theorem 5-5: Converse of the Angle Bisector Theorem

If a point is in the interior of an angle and is equidistant from the sides of the angle, then it is on the angle bisector.

Theorem 5-6: Median of a Triangle

The medians of a triangle are concurrent at a point that is two-thirds the distance from each vertex to the midpoint of the opposite side.

Theorem 5-7: Angle Bisector Theorem

Find , , and .

Theorem 5-8: Angle Bisector Theorem

Find and .

Using the Angle Bisector Theorem

Find .

Examples.

1. Applying the Perpendicular Bisector Theorem

Use the map of Washington, D.C. Describe the set of points that are equidistant from the Lincoln Memorial and the Capitol.

The Converse of the Perpendicular Bisector Theorem states: If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

A point that is equidistant from the Lincoln Memorial and the Capitol must be on the Perpendicular Bisector of the segment whose endpoints are the Lincoln Memorial and the Capitol.

2. Using the Angle Bisector Theorem

Find , , and .

The capital must be on the angle bisector of .

Example: Washington, D.C. Describe the set of points that are equidistant from the Lincoln Memorial and the Capitol. The Converse of the Perpendicular Bisector Theorem states: If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

A point that is equidistant from the Lincoln Memorial and the Capitol must be on the Perpendicular Bisector of the segment whose endpoints are the Lincoln Memorial and the Capitol.

Example: Using the Angle Bisector Theorem

Find , , and .

The capital must be on the angle bisector of .

Example: Using the Angle Bisector Theorem

Find , , and .

The capital must be on the angle bisector of .

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Lesson 5-3 Daily Notetaking Guide

Geometry

Finding the Circumcenter
The circumcenter of a triangle is the point of intersection of the perpendicular bisectors of the sides of a triangle. (Theorem 5-6)

Example

Finding the Circumcenter Find the center of the circle that circumscribes \( \triangle XYZ \).

Because \( X \) has coordinates \((3, 4)\) and \( Y \) has coordinates \((1, 7)\), the vertical line that passes through \((1, \frac{11}{2})\) is the equation of the perpendicular bisector of \( XY \).

Because \( X \) has coordinates \((3, 4)\) and \( Z \) has coordinates \((5, 1)\), the horizontal line that passes through \((\frac{9}{2}, 1)\) is the equation of the perpendicular bisector of \( XZ \).

Draw the lines \( y = \frac{11}{2} \) and \( x = \frac{9}{2} \) that intersect at the point \((\frac{9}{2}, \frac{11}{2})\). This point is the center of the circle that circumscribes \( \triangle XYZ \).

Acute Triangle Right Triangle Obtuse Triangle

Altitude is...

Theorem 5-7 states that the medians of a triangle are concurrent at a point from the sides.

The vertices (0, 0), (3, 4), and (5, 1) lie on a circle with center \((\frac{9}{2}, \frac{11}{2})\) and radius \( \frac{5\sqrt{10}}{2} \).

1. Find the center of the circle that you can circumscribe about the triangle with vertices \((0, 0), (-6, 0), \) and \((6, 0)\).

2. Critical Thinking: In Example 1, explain why it is not necessary to find the third perpendicular bisector.

3. Using the diagram in Example 3, find \( M \). Check that \( WM = MW = WX \).

4. Is \( \triangle XYZ \) a right triangle, or neither? Explain.

Name_________________________ Class____________________________ Date________________

Geometry lesson 5-3

Daily Notetaking Guide

Lesson 5-4

Inverses, Contrapositives, and Indirect Reasoning

Lesson Objectives
- Write the negation of a statement and the converse and contrapositive of a conditional statement
- Use indirect reasoning

Vocabulary and Key Concepts

Negation, Inverse, and Contrapositive Statements

Symbolic Form

You Read It

Conditional

\( p \rightarrow q \)

If an angle is a straight angle, then its measure is 180.

Negation (of \( p \))

\( \neg p \)

An angle is not a straight angle.

Inverse

\( \neg p \rightarrow \neg q \)

If an angle is not a straight angle, then its measure is not 180.

Contrapositive

\( \neg q \rightarrow \neg p \)

If an angle’s measure is not 180, then it is not a straight angle.

Writing an Indirect Proof

Step 1 State an assumption that is contrary to what you want to prove.

Step 2 Show that this assumption leads to a contradiction.

Step 3 Conclude that the assumption must be false, and that what you want to prove must be true.

The negation of a statement has the opposite truth value from that of the original statement.

The inverse of a conditional statement has the same truth value as the converse of the original statement.

The contrapositive of a conditional statement has the same truth value as the converse of the original statement.

An indirect proof is a proof involving indirect reasoning.

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Geometry lesson 5-4

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Examples

1. Writing the Negation of a Statement: Write the negation of "A, B, C, and D are a convex polygon."
   - The negation of a statement is the opposite. To write the negation of a statement, negate both the hypothesis and the conclusion.
   - Hypothesis: A, B, C, and D are a convex polygon.
   - Conclusion: It is not true that A, B, C, and D are a convex polygon.
   - Negation: A, B, C, and D are not a convex polygon.

2. Writing the Inverse and Contrapositive: Write the inverse and contrapositive of the conditional statement "If D is obtuse, then it is isosceles." Then negate both.
   - Hypothesis: D is obtuse.
   - Conclusion: D is isosceles.
   - Inverse: If D is not obtuse, then it is not isosceles.
   - Contrapositive: If D is not isosceles, then it is not obtuse.

3. Writing the Negation of "Each pair of statements to see whether they contradict each other.
   - Two statements contradict each other when they cannot both be true at the same time.
   - Two statements cannot both be true at the same time. Examine each pair of statements to see whether they contradict each other.
   - I. Two segments can be parallel as well as perpendicular.
   - II. Two segments cannot be parallel as well as perpendicular.
   - III. Two segments are perpendicular and congruent.
   - IV. Two segments are parallel and congruent.
   - V. Two segments cannot be parallel as well as perpendicular.
   - VI. Two segments cannot be perpendicular as well as parallel.
   - VII. Two segments cannot be perpendicular as well as congruent.
   - VIII. Two segments cannot be parallel as well as congruent.
   - IX. Two segments are perpendicular and congruent.
   - X. Two segments are parallel and congruent.

Identifying Contradictions: Identify the two statements that contradict each other.
- 1. P, Q, and R are collinear.
- 2. P, Q, and R are non-collinear.
- 3. m\(\angle PQR = 90\)

Two statements contradict each other when they cannot both be true at the same time. Here, statements and pairs of statements to see whether they contradict each other.
- I and II
- I and III
- II and III

Three points that lie on the same line are both collinear and opposite angles. These are two statements that contradict each other.
- I and II
- I and III
- II and III

Three points that lie on an angle are opposite angles. These two statements do not contradict each other.
- I and II
- I and III
- II and III

Indirect Proof: Write an indirect proof.
- Prove: A triangle cannot contain two right angles.
- Assume as true the conclusion of what you want to prove.
- This contradicts the Triangle Angle-Sum Theorem, which states that the sum of the measures of the three angles in a triangle is 180 degrees.
- Therefore, A triangle cannot contain two right angles.

Quick Check:
- a. Write the inverse of the statement "If \(x = y\), then \(x + z = y + z\)."
- b. Today is not Tuesday.

Lesson 5-5: Inequalities in Triangles

Inequalities in Triangles

Lesson Objectives
- Use inequalities involving angles of triangles
- Use inequalities involving sides of triangles

NAEP 2005 Strand: Geometry
- Topic: Relationships Among Geometric Figures
- Local Standards: ____________________

Key Concepts

Comparison Property of Inequality
- If \(a > b\), then \(a + c > b + c\) and \(a - c > b - c\).

Corollary to the Triangle Exterior Angle Theorem
- The measure of an exterior angle of a triangle is greater than the measure of each of its remote interior angles.
- \(m\angle X > m\angle A\) and \(m\angle X > m\angle B\)

Theorem 5-10
- If two sides of a triangle are not congruent, then the larger angle lies opposite the longer side.
- \(\angle A > \angle B\) and \(\angle A > \angle C\)

Theorem 5-11
- If two angles of a triangle are not congruent, then the longer side lies opposite the larger angle.
- \(\angle A > \angle B\) and \(\angle A > \angle C\)

Theorem 5-12: Triangle Inequality Theorem
- The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
- \(LM + MN > LN\) and \(LM + LN > MN\) and \(MN + LN > LM\)
Example

1. Applying the Corollary Explain why \( m \angle 4 > m \angle 5 \).

2. In \( \triangle XYZ \), the \( \angle Y \) is the greater angle of \( \triangle XYZ \). If the Corollary to the Exterior Angle Theorem states that the measure of an exterior angle of a triangle is greater than the measure of each of its remote interior angles, find the measure of each of the angles in \( \triangle XYZ \).

3. Using the Triangle Inequality Theorem Can a triangle have sides with the following lengths?

   - Yes
   - No

4. In parallelogram \( RGYF \), \( RY = 14 \), \( GY = 12 \), and \( BF = 20 \). Find the angle opposite each side.

   - Opposite the longest side.
   - Opposite the shortest side.

5. Define and classify special types of parallelograms with both pairs of opposite sides parallel.

   - Squarerectanglerhombus
   - Local Standards: ____________________________

6. Classifying Quadrilaterals

   - List the angles of \( \triangle ABC \).
   - Find the slope of each side.

   - \( \triangle ABC \) is a trapezoid whose bases are given lengths. Explain.

   - One pair of opposite sides is parallel, so \( \triangle QHM \) is a trapezoid.

   - The slopes of \( \triangle ABC \) are parallel.

   - The slopes of \( \triangle ABC \) are not parallel.

   - The slopes of \( \triangle ABC \) are equal.

   - The slopes of \( \triangle ABC \) are not equal.

   - The slopes of \( \triangle ABC \) are equal and for each one they are parallel.

   - The slopes of \( \triangle ABC \) are not equal and for each one they are parallel.

   - Given \( \triangle ABC \) is an equilateral triangle.

   - \( \triangle ABC \) is a parallelogram.

   - A parallelogram is a quadrilateral with exactly one pair of congruent sides.

   - A parallelogram is a quadrilateral with four congruent sides.

   - A rectangle is a quadrilateral with four right angles.

   - A rhombus is a quadrilateral with four congruent sides.

   - A square is a quadrilateral with four congruent sides and four right angles.

   - Using the Properties of Special Quadrilaterals In \( \square ABCD \), two pairs of opposite sides are congruent, so \( \triangle ABC \) is a parallelogram.

   - Given \( \square ABCD \) is a parallelogram.

   - Given a quadrilateral is parallelogram.

   - In \( \square ABCD \), two pairs of opposite sides are parallel, so \( \angle ABC \) and \( \angle CDA \) are supplementary.

   - \( \angle ABC \) is an exterior angle of \( \triangle ABC \).

   - If two angles are supplementary, then the sum of their measures is 180 degrees.

   - \( m \angle ABC = m \angle CDA \).

   - In \( \triangle ABC \), the measure of an exterior angle is greater than the measure of either of its remote interior angles.

   - The measure of an exterior angle is greater than the measure of any interior angle of the triangle.

   - The measure of an exterior angle is less than the measure of any interior angle of the triangle.
1. a. Graph quadrilateral

Using Algebra
Find the values of the variables in the rhombus. Then find the lengths of the sides.

Using Theorem 6-4
So

Which name gives the most information about the figure? Explain.

Using Algebra
Find the values of the variables in the parallelogram. Then find the lengths of the sides.

It is a rectangle because it has four right angles and its opposite sides are congruent.

A parallelogram because both pairs of opposite sides are congruent.

A square, since its sides are all congruent and it has four right angles.

Using Algebra
Find the values of the variables in the rhombus. Then find the lengths of the sides.

Using Theorem 6-4
So

The diagonals of a parallelogram bisect each other. Use relationships involving diagonals among angles of parallelograms to find the value of each variable.

Theorem 6-3
Opposite angles of a parallelogram are congruent.

Theorem 6-2
Theorem 6-1
Opposite sides of a parallelogram are congruent.

Consecutive angles of a polygon are supplementary. If three (or more) parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on any other transversal. Use relationships among sides and angles of parallelograms to find the value of each variable.

Theorem 6-4
The diagonals of a parallelogram are congruent. Use relationships involving diagonals among angles of parallelograms to find the value of each variable.

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### Lesson 6-3

**Lesson Objective**
- Develop whether a quadrilateral is a parallelogram

**Quick Check.**

1. **Find values for parallelograms**

   - Theorem 6-5
   - If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

   - Theorem 6-6
   - If both pairs of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.

   - Theorem 6-7
   - If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

   - Theorem 6-8
   - Diagonals of parallelograms bisect each other.

2. **Special Parallelograms**

   - Rhombuses
   - Theorem 6-9
   - Each diagonal of a rhombus bisects two angles of the rhombus.

   - Rectangles
   - Theorem 6-11
   - The diagonals of a rectangle are congruent.

   - Parallelograms
   - Theorem 6-12
   - If one diagonal of a parallelogram bisects two angles of the parallelogram, then the parallelogram is a rhombus.

   - Theorem 6-13
   - If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

   - Theorem 6-14
   - If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.
Vocabulary and Key Concepts.

1. Quick Check. Examples.

Theorem 6-10 states that each diagonal of a rhombus bisects two angles of the rhombus so \( m \angle 1 = 70 \).

Theorem 6-9 states that each diagonal of a rhombus is perpendicular.

Because the four angles formed by the diagonals all must have measure 90, \( m \angle 2 = 90 \).

Finally, because \( BE = 90 \), the Triangle Sum Theorem allows you to conclude that \( m \angle 3 = 90 \).

Theorem 6-11 states that the diagonals of a rectangle are congruent.

Because the bases of a trapezoid are parallel, the two angles that share a leg are supplementary.

The base angles of a trapezoid are congruent.

Two angles that share a base of the trapezoid are supplementary.

Quick Check. Examples.

1. Find the measures of the numbered angles in the rhombus.
   \[ m \angle 1 = 90, m \angle 2 = 90, m \angle 3 = 90, m \angle 4 = 90 \]

Lesson 6-5 Daily Notetaking Guide

Finding Angle Measures in Trapezoids

Example 1

Finding Angle Measures in Trapezoids

Example 2

Identifying Special Parallelograms

Example 3

Identifying Special Parallelograms

Example 4

Using Inosculating Trisectors

Example 5

Using Inosculating Trisectors

Quick Check.

1. Find the measures of the numbered angles in the rhombus.

2. Find the length of each diagonal.

3. Find the length of the diagonals of rectangle \( \text{ABCD} \).

4. Find the length of each diagonal of \( \text{WXYZ} \).
Example

Finding Angle Measures in Kites

Find \(m\angle 1, m\angle 2, \) and \(m\angle 3\) in the kite.

The measure of both outer angles is 95.

\[ m\angle 1 = 95 \]
\[ m\angle 2 = 95 \]

Diagonals of a kite are perpendicular.

Thus, the measure of each angle is 47.5.

\[ m\angle 3 = 47.5 \]

Quick Check

1. Find \(m\angle 1, m\angle 2, \) and \(m\angle 3\) in the kite.
   
   \[ m\angle 1 = 80, \quad m\angle 2 = 46, \quad m\angle 3 = 46 \]

2. Find \(m\angle 1, m\angle 2, \) and \(m\angle 3\) in the kite.
   
   \[ m\angle 1 = 75, \quad m\angle 2 = 50, \quad m\angle 3 = 50 \]

Lesson 6-6

Placing Figures in the Coordinate Plane

Lesson Objective

Type: Proving Congruency

Topic: Position and Direction

Local Standards: 12.04.01.01

Example

Proving Congruency

Show that \(TWVU\) is a parallelogram by proving pairs of opposite sides congruent.

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram by Theorem 6-18.

You can prove that \(TWVU\) is a parallelogram by showing that \(TW = UV\) and \(TV = UW\) by the distance formula.

Use the coordinates of \(T(2, 0), \quad W(-2, 2), \quad V(6, 0), \) and \(U(-6, -2)\).

\[ TW = \sqrt{(2 - (-2))^2 + (0 - 2)^2} = \sqrt{4^2 + 2^2} = \sqrt{20} \]
\[ TV = \sqrt{(6 - 2)^2 + (0 - 0)^2} = \sqrt{4^2 + 0^2} = 4 \]
\[ UW = \sqrt{(-6 - (-2))^2 + (-2 - 2)^2} = \sqrt{(-4)^2 + (-4)^2} = \sqrt{32} \]
\[ UV = \sqrt{(6 - (-2))^2 + (0 - (-2))^2} = \sqrt{8^2 + 2^2} = \sqrt{68} \]

Because \(TW = UV\) and \(TV = UW\), \(TWVU\) is a parallelogram.

Quick Check

1. Use the diagram above. Use a different method: Show that \(TRAP\) is a parallelogram by finding the midpoints of the diagonals.

   Midpoint of \(TP = \left(\frac{2+(-2)}{2}, \frac{0+2}{2}\right) = (0, 1)\) is midpoint of \(TR\).

   Thus, the diagonals bisect each other, and \(TRAP\) is a parallelogram.

Example

Planning a Coordinate Geometry Proof

Examine trapezoid \(TRAP\). Explain why you can assign the same \(y\)-coordinate to points \(A\) and \(A\).

In a trapezoid, only one pair of sides is parallel. In \(TRAP\), \(PP' = \overline{TT'}\). Because \(PP'\) lies on the horizontal line \(x = 2\), \(TT'\) also lies on the same horizontal line as \(PP'\).

The \(y\)-coordinate of all points on a horizontal line are the same, so points \(B\) and \(D\) have the same \(y\)-coordinate.
Geometry: All-In-One Answers Version A (continued)

Lesson 7-1  Ratios and Proportions

Name_____________________________________ Class____________________________ Date________________

1. A scale model of a car is 4 in. long. The actual car is 15 ft long. What is the ratio of the length of the model to the length of the car?

2. Write two expressions that are equivalent to .

3. Find and compare the lengths and .

4. In parts (i) and (ii), how does placing a base along the x-axis help?

Quick Check.

1. A photo that is 8 in. wide and 12 in. high is enlarged to a poster that is 2 ft wide and 3 ft high. What is the ratio of the height of the photo to the height of the poster?

2. Write two expressions that are equivalent to .

All-In-One Answers Version A
Quick Check.

1. Solve each proportion.
   a. \( \frac{3}{5} = \frac{6}{x} \)
   b. \( \frac{4}{7} = \frac{z}{5} \)
   c. \( \frac{2}{3} = \frac{8}{21} \)
   d. \( \frac{1}{2} = \frac{y}{10} \)

2. The cities are actually 175 miles apart. What would be the distance in inches between the cities on the new map?
   a. Yes; corresponding angles are congruent and corresponding sides are proportional.
   b. \( \frac{30}{5} \) corresponds to \( \frac{75}{25} \) in., so \( \frac{30}{75} = \frac{5}{25} \)

3. Sketch \( \triangle ABC \) and \( \triangle XYZ \).
   a. Yes; corresponding angles are congruent and corresponding sides are proportional.
   b. \( \frac{5}{10} \) corresponds to \( \frac{5}{10} \) in., so \( \frac{5}{10} = \frac{1}{2} \)

4. Recall Example 4. You want to make a new map with a scale of 1 in. = 50 mi. Two cities that are actually 175 miles apart are to be represented on your map. What would be the distance in inches between the cities on the new map?

   a. \( \frac{175}{50} \) inches

Lesson 7-2 Similar Polygons

Lesson Objectives
- Identify similar polygons
- Apply similar polygons
- Use proportions to solve problems

Vocabulary
- Similar figures: figures that have the same shape but not necessarily the same size. Two polygons are similar if corresponding angles are congruent and corresponding sides are proportional.
- The mathematical symbol for similarity is \( \sim \).

The similarity ratio is the ratio of the lengths of corresponding sides of similar figures.

A golden rectangle is a rectangle that can be divided into a square and a rectangle that is similar to the original rectangle.

The golden ratio, \( \phi \), is the ratio of the length to the width of any golden rectangle, about 1.618 : 1.

Example

1. Understanding Similarity
   \( \triangle ABC \sim \triangle XYZ \)
   a. \( \angle A = \angle X \) and \( \angle C = \angle Z \), so \( \angle B = \angle Y \) because corresponding angles have the same measure.
   b. \( \frac{BC}{YX} = \frac{2}{1} \)

Quick Check

1. Refer to the diagram for Example 1. Complete:
   a. \( \frac{AC}{XY} = \frac{3}{2} \)
   b. \( \frac{AB}{XZ} = \frac{2}{3} \)

2. Match \( \triangle XYZ \) and \( \triangle MNP \) with \( \angle X = \angle M \), \( \angle Y = \angle N \), and \( \angle Z = \angle P \).

   a. \( \angle Y = \angle Z \)
   b. \( \angle X = \angle M \)
   c. \( \angle Z = \angle P \)
   d. \( \angle Y = \angle Z \)

X. Refer to the diagram for Example 3. Find \( AR \).

   \( \frac{AR}{16} = \frac{8}{12} \)

4. A golden rectangle has shorter sides of length 30 cm. Find the length of the longer sides.

   a. 48 cm
   b. 60 cm
   c. 50 cm
   d. 70 cm

   The golden ratio is about 1.62.

Lesson 7-1 Daily Notetaking Guide

Geometry: All-In-One Answers Version A (continued)

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Lesson 7-3
Proving Triangles Similar

Vocabulary and Key Concepts

Postulate 7-1: Angle-Angle Similarity (AA~) Postulate
If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

Theorem 7-1: Side-Angle-Side Similarity (SAS~) Theorem
If an angle of one triangle is congruent to an angle of a second triangle, and the sides including the two angles are proportional, then the triangles are similar.

Theorem 7-2: Side-Side-Side Similarity (SSS~) Theorem
If the corresponding sides of two triangles are proportional, then the triangles are similar.

Examples

1. Using the AA~ Postulate
Explain why the triangles are similar. Write a similarity statement.

2. Using Similarity Theorems
Explain why the triangles must be similar. Write a similarity statement.

3. Explain why the triangles must be similar. Write a similarity statement.

4. Finding the Geometric Mean
Find and use relationships in similar right triangles.

Local Standards: ____________________________________

Lesson 7-4
Similarity in Right Triangles

Vocabulary and Key Concepts

Theorem 7-3
The altitude to the hypotenuse of a right triangle divides the triangle into two triangles that are similar to the original triangle and to each other.

Corollary 1 to Theorem 7-3
The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the segments of the hypotenuse.

Corollary 2 to Theorem 7-3
The altitude to the hypotenuse of a right triangle separates the hypotenuse so that the length of each leg of the triangle is the geometric mean of the lengths of the segments of the hypotenuse and the length of the hypotenuse.

The geometric mean of two positive numbers a and b is the positive number x such that

\[ ax = xb \]

Examples

Finding the Geometric Mean
Find the geometric mean of 3 and 12.

1. Find the geometric mean of 3 and 12.

Local Standards: ____________________________________

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Finding Distance: At a golf course, Maria drove her ball 192 yd straight toward the cup. Her brother Gabriel drove his ball straight 240 yd, but not toward the cup. The diagram shows the results. Find $x$ and $y$, their remaining distances from the cup.

Use Corollary 2 of Theorem 7-3 to solve for $x$.

Find the geometric mean of 15 and 20.

Recall Example 2. Find the distance between Maria’s ball and Gabriel’s ball.

Quick Check.

1. Find the geometric mean of 15 and 20.

2. Solve for $x$.

Find the positive square root.

Write a proportion.

Cross-Product Property

María’s ball is $x$ yd from the cup, and Gabriel’s ball is $y$ yd from the cup.

Quick Check.

1. Find the geometric mean of 15 and 20.

2. Solve for $x$.

Recall Example 2. Find the distance between Maria’s ball and Gabriel’s ball.

Quick Check.

1. Find the geometric mean of 15 and 20.

2. Solve for $x$.

Maria’s ball is $x$ yd from the cup, and Gabriel’s ball is $y$ yd from the cup.

Quick Check.

1. Find the geometric mean of 15 and 20.

2. Solve for $x$.

Recall Example 2. Find the distance between Maria’s ball and Gabriel’s ball.
Lesson 8-1 The Pythagorean Theorem and Its Converse

Vocabulary and Key Concepts

**Example:** 2

- **Theorem 8-1: Pythagorean Theorem**
  - In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

- **Theorem 8-2: Converse of the Pythagorean Theorem**
  - If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.

- **Pythagorean Triple**
  - A set of nonzero whole numbers (a, b, c) that satisfy $a^2 + b^2 = c^2$.

- **Simplest Radical Form**
  - An answer in simplest radical form means that the answer is simplified as much as possible.

- **Local Standards:**
  - NAEP 2005 Strand: Relationships Among Geometric Figures

Lesson Objectives

- Use the Pythagorean Theorem
- Use the Converse of the Pythagorean Theorem

Quick Check

1. A right triangle has a hypotenuse of length 25 and a leg of length 10. Find the length of the other leg. Do the lengths of the sides form a Pythagorean triple?

2. The length of the hypotenuse of $\triangle ABC$ is 13. The lengths of the sides are 5, 12, and 13. Is $\triangle ABC$ a Pythagorean triple?

Quick Check

1. A right triangle has a hypotenuse of length 25 and a leg of length 10. Find the length of the other leg. Do the lengths of the sides form a Pythagorean triple?

2. The length of the hypotenuse of $\triangle ABC$ is 13. The lengths of the sides are 5, 12, and 13. Is $\triangle ABC$ a Pythagorean triple?
Lesson 8-2

Special Right Triangles

Example 1
Finding the Length of the Hypotenuse: Find the value of the variable. Use the 45°-45°-90° Triangle Theorem to find the hypotenuse.

\[ \text{h} = 5 \sqrt{2} \]

Example 2
Applying the 30°-60°-90° Triangle Theorem: An equilateral triangle has a side length of 10 in. Find the area of the triangle.

\[ \text{Area} = \frac{\sqrt{3}}{4} \times (10)^2 = 25 \sqrt{3} \text{in}^2 \]

Quick Check
1. Find the length of the hypotenuse of a 45°-45°-90° triangle with legs of length 5.5.

2. A square garden has sides of 100 ft long. You want to build a brick path along a diagonal of the square. How long will the path be? Round your answer to the nearest foot.

Lesson 8-3

The Tangent Ratio

Example 1
Writing Tangent Ratios: Write the tangent ratios for \( \angle A \) and \( \angle B \).

\[ \tan A = \frac{BC}{AC} \quad \tan B = \frac{AC}{BC} \]

Quick Check
1. A rhombus has sides of 10 in. Find the area of each rhombus.

2. A square garden has sides of 100 ft long. You want to build a brick path along a diagonal of the square. How long will the path be? Round your answer to the nearest foot.
Lesson 8-4 Sine and Cosine Ratios

**Objective:**
- Use sine and cosine to determine side lengths in triangles
- NAEP 2005 Strand: Measurement
- Topic: Measuring Physical Attributes
- Local Standards:

**Vocabulary:**
- The sine of \( \angle A \) is the ratio of the length of the leg opposite \( \angle A \) to the length of the hypotenuse.
- The cosine of \( \angle A \) is the ratio of the length of the leg adjacent to \( \angle A \) to the length of the hypotenuse.
- The sine of \( \angle A \) can be abbreviated as \( \sin A \).
- The cosine of \( \angle A \) can be abbreviated as \( \cos A \).

**Examples:**

**Writing Sine and Cosine Ratios:** Use the triangle to find \( \sin T \), \( \cos T \), \( \sin G \), and \( \cos G \). Write your answers to simplest form.

- \( \sin T = \frac{opposite}{hypotenuse} \)
- \( \cos T = \frac{adjacent}{hypotenuse} \)
- \( \sin G = \frac{opposite}{hypotenuse} \)
- \( \cos G = \frac{adjacent}{hypotenuse} \)

**Using the Sine Ratio:** To measure the height of a tree, Alene walked 125 ft from the tree and measured a 32° angle from the ground to the top of the tree. Estimate the height of the tree.

- \( \tan \theta = \frac{opposite}{adjacent} \)
- \( \text{height} = 125 \times \tan 32° \)
- \( \text{height} \approx 78.108669 \)

**Using the Cosine Ratio:** A 20-ft wire supporting a flagpole forms a 35° angle with the flagpole. To the nearest foot, how high is the flagpole?

- \( \cos \theta = \frac{adjacent}{hypotenuse} \)
- \( \text{height} = 20 \times \cos 35° \)
- \( \text{height} \approx 13.8 \)

**Using the Inverse of Sine and Cosine:** A right triangle has a leg 1.5 units long and hypotenuse 4.0 units long. Find the measure of its acute angles to the nearest degree.

- **Using the Inverse of Sine:**
  - \( \sin \theta = \frac{opposite}{hypotenuse} \)
  - \( \theta = \sin^{-1} \left( \frac{opposite}{hypotenuse} \right) \)
  - \( \theta \approx 100° \)
- **Using the Inverse of Cosine:**
  - \( \cos \theta = \frac{adjacent}{hypotenuse} \)
  - \( \theta = \cos^{-1} \left( \frac{adjacent}{hypotenuse} \right) \)
  - \( \theta \approx 22° \)

**Writing Sine and Cosine Ratios:** Use the triangle to find \( \sin T \), \( \cos T \), \( \sin G \), and \( \cos G \). Write your answers to simplest form.

- \( \sin T = \frac{opposite}{hypotenuse} \)
- \( \cos T = \frac{adjacent}{hypotenuse} \)
- \( \sin G = \frac{opposite}{hypotenuse} \)
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  - \( \theta \approx 100° \)
- **Using the Inverse of Cosine:**
  - \( \cos \theta = \frac{adjacent}{hypotenuse} \)
  - \( \theta = \cos^{-1} \left( \frac{adjacent}{hypotenuse} \right) \)
  - \( \theta \approx 22° \)
Quick Check.

1. a. Write the sine and cosine ratios for $\angle X$ and $\angle Y$.
   
   \[
   \sin X = \frac{opposite}{hypotenuse} = \frac{40}{50} = 0.8 \quad \cos X = \frac{adjacent}{hypotenuse} = \frac{30}{50} = 0.6
   \]

   b. In general, how are $\sin X$ and $\cos Y$ related? Explain.
   
   $\sin X = \cos Y$ when $\angle X$ and $\angle Y$ are complementary.

2. In Example 2, suppose that the angle the wire makes with the ground is 50°. What is the height of the flagpole to the nearest foot?

   The building is about 200 ft tall.

3. Find the value of $x$. Round your answer to the nearest degree.

   a. $\tan x = \frac{opposite}{adjacent} = \frac{3}{4}$
      
      $x = \tan^{-1} \left( \frac{3}{4} \right) \approx 36.9°$
      
      b. $\tan x = \frac{opposite}{adjacent} = \frac{10}{15}$
      
      $x = \tan^{-1} \left( \frac{10}{15} \right) \approx 31.0°$

4. You sight a rock climber on a cliff at a 32° angle of elevation. The horizontal distance to the cliff is 4800 ft. How tall is the cliff?

   $\tan 32° = \frac{opposite}{adjacent} = \frac{h}{4800}$
   
   $h = 4800 \times \tan 32° \approx 2540$ ft

5. An airplane flying 3500 ft above the ground begins a 2° descent. The horizontal ground distance to the cliff is 10000 ft. Find the line-of-sight distance to the cliff.

   $\tan 2° = \frac{opposite}{adjacent} = \frac{h}{10000}$
   
   $h = 10000 \times \tan 2° \approx 343$ ft

6. You sight a life raft at a 26° angle of depression. The airplane's altitude is 3 km. What is the airplane's horizontal distance from the raft?

   $\tan 26° = \frac{opposite}{adjacent} = \frac{3000}{x}$
   
   $x = \frac{3000}{\tan 26°} \approx 6280$ ft

7. The angle of elevation from the building to the person on the ground

   a. $\angle 1$
      
      \[ \angle 1 = \angle 3 \]

   b. $\angle 2$
      
      \[ \angle 2 = \angle 4 \]

8. You sight a rock climber on a cliff at a 37° angle of elevation. The horizontal ground distance to the cliff is 1000 ft. Find the line-of-sight distance to the rock climber.

   $\tan 37° = \frac{opposite}{adjacent} = \frac{h}{1000}$
   
   $h = 1000 \times \tan 37° \approx 727$ ft

9. An airplane pilot sees a life raft at a 20° angle of depression. The airplane's altitude is 3 km. What is the airplane's horizontal distance from the raft?

   $\tan 20° = \frac{opposite}{adjacent} = \frac{3000}{x}$
   
   $x = \frac{3000}{\tan 20°} \approx 8790$ ft

Lesson 8-5 Daily Notetaking Guide

Geometry

- Identify angles of elevation and depression to solve problems.
- Use right triangles and trigonometric ratios to solve problems involving heights and distances.

Identifying Angles of Elevation and Depression

Describe $\angle 1$ and $\angle 2$ as they relate to the situation shown.

- One side of the angle of depression is a horizontal line. $\angle 2$ is the angle of elevation from the building to the airplane.
- One side of the angle of elevation is a horizontal line. $\angle 1$ is the angle of depression from the building to the person on the ground.

Check Your Understanding

1. Describe each angle as it relates to the situation in Example 1.

   a. $\angle 3$
      
      The angle of depression from the building to the person on the ground.

   b. $\angle 4$
      
      The angle of elevation from the person on the ground to the building.

2. You sight a rock climber on a cliff at a 37° angle of elevation. The horizontal ground distance to the cliff is 1000 ft. Find the line-of-sight distance to the rock climber.

   $\tan 37° = \frac{opposite}{adjacent} = \frac{h}{1000}$
   
   $h = 1000 \times \tan 37° \approx 727$ ft

3. An airplane pilot sees a life raft at a 20° angle of depression. The airplane's altitude is 3 km. What is the airplane's horizontal distance from the raft?

   $\tan 20° = \frac{opposite}{adjacent} = \frac{3000}{x}$
   
   $x = \frac{3000}{\tan 20°} \approx 8790$ ft
Lesson 8-6

**Vectors**

**Lesson Objectives**
- Describe vectors
- Solve problems that involve vector addition

**Vocabulary and Key Concepts**

A vector is any quantity with magnitude (size) and direction.

A vector can be represented with an arrow.

The magnitude of a vector is the size or length.

The initial point of a vector is the point at which it starts.

The terminal point of a vector is the point at which it ends.

A constant vector is the sum of other vectors.

The magnitude of a vector \( \mathbf{v} \) is the distance from its initial point to its terminal point.

The ordered pair \( (x_1, y_1) \) is the position of the initial point of a vector.

The ordered pair \( (x_2, y_2) \) is the position of the terminal point of a vector.

The resultant vector is the sum of two or more vectors.

The terminal point of a vector is the point at which it ends.

The initial point of a vector is the point at which it starts.

A vector can be represented with an arrow.

To find the direction a boat sails, find the angle that the vector forms with the \( x \)-axis.

The trip can be described by the vector \( \mathbf{v} \).

Use distance and direction to describe the vector a second way.

Draw a diagram for the situation.

To find the distance sailed, use the Distance Formula.

The magnitude of the vector is the distance from the initial point to the terminal point.

The vector \( \mathbf{v} \) is given by the ordered pair \( (x, y) \).

Add the coordinates.

Simplify.

Find the angle that the vector forms with the \( x \)-axis.

Use the tangent ratio.

Take the square root.

Use a calculator.

Find the angle whose tangent is 0.75.

Find the angle whose tangent is 4.25.

The boat sailed about 270 mi south of \( S \).

**Examples**

**Describing a Vector**

Describe the vector \( \mathbf{v} \) as an ordered pair.

Use sine and cosine.

Because point \( M \) is in the \( Q \) quadrant, both coordinates are negative.

To the nearest tenth, \( \mathbf{v} \) is about -4.0, -5.0.

**Describing a Vector Direction**

A boat sailed 12 mi east and 9 mi south.

The trip can be described by the vector \( \mathbf{v} \). Use distance and direction to describe the vector a second way.

Draw a diagram for the situation.

To find the distance sailed, use the Distance Formula.

The magnitude of the vector is the distance from the initial point to the terminal point.

The vector \( \mathbf{v} \) is given by the ordered pair \( (x, y) \).

Add the coordinates.

Simplify.

Find the angle that the vector forms with the \( x \)-axis.

Use the tangent ratio.

Use sine and cosine.

Because point \( M \) is in the \( Q \) quadrant, both coordinates are negative.

To the nearest tenth, \( \mathbf{v} \) is about -4.1, -5.1.

**Quick Check**

1. Describe the vector at the right as an ordered pair. Give the coordinates to the nearest tenth.

2. A small airplane lands 126 mi west and 70 mi south of the point from which it took off. Describe the magnitude and the direction of the right vector.

3. Write the sum of the two vectors \((2, 3)\) and \((-4, -2)\) as an ordered pair.

4. Simplify.

5. Find the angle whose tangent is 0.75.

6. Find the angle whose tangent is 4.25.

7. The boat sailed about 270 mi south of \( S \).
Geometry: All-In-One Answers Version A (continued)

Name ______________________ Class __________________ Date ______________

**Quick Check.**

1. What are the coordinates of the image of \( Q \) if the reflection line is the perpendicular bisector of \( \overline{AC} \)?
   - \( Q' \) is at \((-1, -5)\)

2. Use the rule \((x, y) \rightarrow (x + 3, y - 1)\) to find the translation image of \( \triangle LMN \).
   - Graph the image \( \triangle L'M'N' \)

3. Finding a Translation Image
   - Find the image of \( \triangle ABC \) for the translation \((x, y) \rightarrow (x - 2, y + 3)\).
   - \( A'(-4, 1), B'(2, 4), C'(1, 2) \)

4. Does the transformation appear to be an isometry? Explain.
   - Yes; the figures appear to be congruent by a flip.

5. Find the image of \( \triangle IJK \) for the translation \((x, y) \rightarrow (x - 2, y + 3)\).
   - \( I'(2, -1), J'(4, 1), K'(3, 4) \)

**Lesson 9-2 Reflections**

**Lesson Objectives**

- Find reflection images of figures

**Vocabulary**

- A reflection in line \( l \) is a transformation such that if \( A \) is on line \( l \), then the image of \( A \) is \( A' \) and if a point \( B \) is not on line \( l \), then its image \( B' \) is the point such that \( \overline{AB} \parallel \overline{AB'} \)

**Example 1**

- **Finding Reflection Images:** If point \( P(2, -1) \) is reflected across line \( x = 1 \), what are the coordinates of its reflection image?
  - \( P' \) is at \((2, 2)\)

**Quick Check 1**

- What are the coordinates of the image of \( Q' \) if the reflection line is the perpendicular bisector of \( \overline{AC} \)?
  - \((0, -4)\)

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**Lesson 9-2 Reflections**

**Lesson Objectives**

- Find reflection images of figures

**Vocabulary**

- A reflection in line \( l \) is a transformation such that if \( A \) is on line \( l \), then the image of \( A \) is \( A' \) and if a point \( B \) is not on line \( l \), then its image \( B' \) is the point such that \( \overline{AB} \parallel \overline{AB'} \)

**Example 1**

- **Finding Reflection Images:** If point \( Q(-3, -1) \) is reflected across line \( x = -1 \), what are the coordinates of its reflection image?
  - \( Q' \) is at \((-2, 1)\)

**Quick Check 1**

- What are the coordinates of the image of \( Q' \) if the reflection line is the perpendicular bisector of \( \overline{AC} \)?
  - \((0, -4)\)
Lesson 9-3

Rotations

Example 1

Drawing a Rotation Image

Draw the image of \( \triangle ABC \) under a 60\(^\circ\) rotation about point \( P \).

Example 2

Finding an Angle of Rotation

A regular 12-sided polygon can be formed by stacking congruent square sheets of paper rotated about the same center on top of each other. Find the angle of rotation about point \( M \) that maps \( W \) to \( B \).

Quick Check

1. Draw the image of \( \triangle LOB \) for a 90\(^\circ\) rotation about point \( B \). Label the vertices of the image.

2. Regular pentagon \( \text{PENTA} \) is divided into 5 congruent triangles. Name the image of \( \text{T} \) for a 144\(^\circ\) rotation about point \( E \).

Example 3

Compositions of Rotations

Describe the image of \( \triangle XYZ \) under a composition of a 140\(^\circ\) rotation and then a 215\(^\circ\) rotation about point \( X \).

Quick Check

1. In the figure from Example 3, find the angle of rotation about point \( M \) that maps \( \text{R} \) to \( \text{K} \).

2. Draw the image of the kite for a composition of two 90\(^\circ\) rotations about point \( K \).

Vocabulary

A rotation of a point \( A \) about a point \( B \) is a transformation for which the following are true:

- The image of \( A \) in a rotation is called the rotation image in the line \( AB \).
- The rotation image of \( AB \) is the same as \( BA \).
- The rotation image of \( A \) with one side \( AB \) is the point \( B \).
Lesson 9-4 Symmetry

Vocabulary

A figure has symmetry if there is an isometry that maps the figure onto itself.

A figure has rotational symmetry if it has 180° rotational symmetry.

A figure has point symmetry if it has 180° rotational symmetry.

A figure has reflectional symmetry if there is a reflection that maps the figure onto itself.

Line symmetry is the same as reflectional symmetry.

A figure has rotational symmetry if it is its own image for some rotation of 180° or less.

The letter V does not have rotational symmetry because it must be rotated yes; 180° before it is its own image.

The nut is its own image after one quarter-turn, so it has rotational symmetry.

The nut has lines of symmetry.

The nut has a square outline with a circular opening. The square and circle faces, like the top view shown below. Tell whether the nut has rotational symmetry and/or reflectional symmetry. Draw all lines of symmetry.

An enlargement is a dilation with a scale factor greater than 1.

A reduction is a dilation with a scale factor less than 1.

As an enlargement is a dilation with a scale factor greater than 1.

A reduction is a dilation with a scale factor less than 1.

Example 1: Identifying Lines of Symmetry

Draw all lines of symmetry for the isosceles trapezoid.

Example 2: Identifying Rotational Symmetry

Judging from appearance, do the letters V and H have rotational symmetry? If so, give an angle of rotation.

The letter V does not have rotational symmetry because it must be rotated yes; 180° before it is its own image.

The letter H is its own image after one half-turn, so it has rotational symmetry with a 180° angle of rotation.

Finding Symmetry

A nut holds a bolt in place. Some nuts have square faces, like the top view shown below. Tell whether the nut has rotational symmetry and/or reflectional symmetry. Draw all lines of symmetry.

The nut has lines of symmetry.

The nut has a square outline with a circular opening. The square and circle faces, like the top view shown below. Tell whether the nut has rotational symmetry and/or reflectional symmetry. Draw all lines of symmetry.

The nut has lines of symmetry.

The nut has a square outline with a circular opening. The square and circle faces, like the top view shown below. Tell whether the nut has rotational symmetry and/or reflectional symmetry. Draw all lines of symmetry.

The nut has lines of symmetry.
Lesson 9-6: Compositions of Reflections

Name _________________________ Class __________________ Date ______________

Quick Check

1. Quadrilateral JKLM is a dilation image of quadrilateral ABCD.
   Describe the dilation.

   The dilation is a reduction with center (0, 0) and scale factor \( \frac{1}{2} \).

2. The height of a tractor-trailer truck is 4.2 m. The scale factor for a model truck is 1:20.
   Find the height of the model to the nearest centimeter.

   \[ \text{Model height} = \frac{1}{20} \times 4.2 \text{ m} \approx 0.21 \text{ m} \]

A. Find the image of \( \triangle ABC \) for a dilation with center \((0, 0)\) and scale factor \( \frac{1}{2} \).

   Draw the dilation on the grid and give the coordinates of the image’s vertices.

   Vertices: \( P(1, 0) \rightarrow P'(0.5, 0) \), \( Q(4, 1) \rightarrow Q'(2, 0.5) \), and \( R(1, -1) \rightarrow R'(0.5, -0.5) \).

Lesson 9-6: Compositions of Reflections

Name _________________________ Class __________________ Date ______________

Lesson Objective:

- Use a composition of reflections.
- Identify glide reflections.

Vocabulary and Key Concepts

Theorem 9-1: A translation or rotation is a composition of two reflections.

Theorem 9-2: A composition of reflections across two parallel lines is a translation.

Theorem 9-3: A composition of reflections across two intersecting lines is a rotation.

The Fundamental Theorem of Isometries

In a plane, one of two congruent figures can be mapped onto the other by a composition of at most three transformations.

There are only four isometries. They are the following:

- Reflection
- Translation
- Rotation
- Glide reflection

A glide reflection is the composition of a glide translation and a reflection across a line parallel to the direction of translation.

Example

A composition of reflections in intersecting lines. The letter D is reflected in line x and then in line y. Describe the resulting rotation.

Find the image of D through a reflection across line x. Then, reflect the translated image across line y.

The composition of two reflections across intersecting lines is a rotation.

The center of rotation is the point where the lines of reflection intersect.

Draw the angle formed by the intersecting lines. In the letter D is oriented clockwise, with the center of rotation at point O.

Finding a Glide Reflection Image

\( \triangle ABC \) has vertices \( A(4, 5) \), \( B(2, 4) \), and \( C(3, 3) \). Find the image of \( \triangle ABC \) for a glide reflection where the translation is \( (x, y) \rightarrow (x + 1, y) \) and the reflection line is \( x = 1 \).

First, translate \( \triangle ABC \) by \( (x, y) \rightarrow (x + 1, y) \):

- \( A(4, 5) \rightarrow A'(5, 5) \)
- \( B(2, 4) \rightarrow B'(3, 4) \)
- \( C(3, 3) \rightarrow C'(4, 3) \)

Then, reflect the translated image across the line \( x = 1 \):
Quick Check

1. (a) Reflect the letter R across a and then b. Describe the resulting rotation.
   A rotation of ( / 3 ) 30°. The result is a clockwise rotation about point P, through an angle of 90°, clockwise.
   
   (b) Use parallel lines f and m. Draw R between f and m. Find the image of R for a reflection across line f and then across line m. Describe the resulting translation.
   A translation 1/2 units right and 1 units up. The result is a translation twice the distance between f and m.

2. (a) Find the image of R under a glide reflection where the translation is (x, y) = (x + 1, y) and the reflection line is y = 2. Draw the translation first, then the reflection.
   (b) Would the result of part (a) be the same if you reflected R onto a copy of itself along any of the lines.
   Yes; Order of steps does not matter in a glide reflection.

Lesson 9-7 Tessellations

Vocabulary and Key Concepts

A tessellation, or tiling, is a repeating pattern of figures that completely covers a plane without gaps or overlaps. The type of symmetry for which there is a translation that maps a figure onto itself is translational symmetry. The type of symmetry for which there is a glide reflection that maps a figure onto itself is glide reflectional symmetry.

Examples

1. Determine whether a regular 15-gon tessellates a plane. Explain.
   Because the measures of the angles around any vertex must sum to 360°, a regular 15-gon will not tessellate a plane.

2. Determine whether a regular 18-gon is a factor of 360. Check to see whether the measure of an angle of a regular 18-gon is a factor of 360.
   a. The measures of the angles around any vertex must sum to 360°.
   b. Check whether the measure of an angle of a regular 18-gon is a factor of 360.

3. Identify symmetries in tessellations.
   Translational symmetry is the type of symmetry for which there is a translation that maps a figure onto itself.
   Glide reflectional symmetry is the type of symmetry for which there is a glide reflection that maps a figure onto itself.

Local Standards: ____________________________________

Topic: Geometry

NAEP 2005 Strand: Geometry

Lesson Objectives

Identify transformation in tessellations and figures that will tessellate.
Identify symmetries in tessellations.

Examples

1. Determine whether a regular 15-gon tessellates a plane. Explain.
   
   a. If the angle measures are not factors of 360, then the tessellation will not tessellate a plane.
   b. Check whether the measure of an angle of a regular 15-gon is a factor of 360.

2. Identify symmetries in tessellations.
   
   a. Translational symmetry
   b. Glide reflectional symmetry
   c. Rotation

3. List the symmetries in the tessellation.
   
   Symmetries: rotational symmetry, glide reflectional symmetry, translational symmetry.
Lesson 10-1
Areas of Parallelograms and Triangles

Quick Check.

1. Find the area of the triangle.

2. Critical Thinking

Suppose the bases of the square and triangle in Example 3 are doubled to 30 ft, but the height of each figure stays the same. How is the force of the wind against the building affected?

Lesson Objectives
- Find the area of a parallelogram
- Find the area of a triangle

Vocabulary and Key Concepts
- A base of a parallelogram is the length of the altitude to the line containing that base.
- The height of a parallelogram is the length of the altitude to the line containing that base.

Example 1

A parallelogram has sides 15 cm and 18 cm. The height corresponding to a 15-cm base is 9 cm. Find the height corresponding to an 18-cm base.

Example 2

A parallelogram has 9-in. and 18-in. sides. Finding a Missing Dimension

The height corresponding to the 9-in. base is 11 in. Find the height corresponding to the 18-in. base.

Example 3

A parallelogram has 9-in. and 18-in. sides. Example 3

Find the area of a trapezoid

The area of a trapezoid is the sum of the areas of its two bases and its height.

Example 4

A trapezoid has bases 15 in. and 16 in. The height corresponding to a 15-in. base is 7 in. Find the height corresponding to a 16-in. base.

Quick Check

1. Find the area of a parallelogram

2. Find the area of a triangle

3. Critical Thinking

Suppose the bases of the square and triangle in Example 3 are doubled to 30 ft, but the height of each figure stays the same. How is the force of the wind against the building affected?

The force is doubled.
Lesson 10-3 Areas of Regular Polygons

Examples

1. In Example 2, suppose \(a = 6\) and \(b = 12\), change so that \(m = \frac{1}{4}\) and \(n = \frac{1}{2}\).
2. In Example 2, suppose \(a = 6\) and \(b = 12\), change so that \(m = \frac{1}{3}\) and \(n = \frac{1}{2}\).

Quick Check

1. Find the area of a trapezoid with height 7 cm and bases 12 cm and 15 cm.

2. In Example 2, suppose \(a = 6\) and \(b = 12\), change so that \(m = \frac{1}{4}\) and \(n = \frac{1}{2}\). Find the area of a trapezoid ABCD.

3. Find the area of kite XYZW with diagonals 12 in. and 9 in. long.

4. Critical Thinking: In Example 4, explain how you can use a Pythagorean triple to conclude that \(XU = 5\).
Quick Check
1. As the right, a portion of a regular octagon has radii and an apothem drawn. Find the measures of each numbered angle.

2. Find the area of a regular pentagon with 11.6-cm sides and an 8-cm apothem.

3. The side of a regular hexagon is 16 ft. Find the area of the hexagon.

4. Two similar polygons have corresponding sides in the ratio 5 : 7.

   a. Find the ratio of their perimeters.
   b. Find the ratio of their areas.

Example
The areas of two similar rectangles are 1875 ft² and 135 ft². Find the ratio of their perimeters and of their areas.

Using Similarity Ratios
Benita plants the same crop in two rectangular fields. Each dimension of the larger field is 2 times the dimension of the smaller field. Seeding the smaller field costs $8. How much money does seeding the larger field cost?

Example
Finding Ratios in Similar Figures: The triangles at the right are similar. Find the ratio (larger to smaller) of their perimeters and of their areas.

Finding Areas Using Similar Figures
The ratio of the length of the corresponding sides of two regular octagons is , the ratio of their areas is , or .

Theorem 10-7: Perimeters and Areas of Similar Figures
If the similarity ratio of two similar figures is , then

1. the ratio of their perimeters is 

2. the ratio of their areas is 

Example
Finding Similarity and Perimeter Ratios: The areas of two similar parallelograms are 52 in² and 72 in². What is their similarity ratio? What is the ratio of their perimeters?

Quick Check
1. Two similar polygons have corresponding sides in the ratio 5 : 7.
   a. Find the ratio of their perimeters.
   b. Find the ratio of their areas.

2. The corresponding sides of two similar parallelograms are in the ratio .
   a. Find the ratio of their perimeters.
   b. Find the area of the smaller parallelogram.

3. The similarity ratio of the dimensions of two similar pieces of window glass is 5 : 3. The smaller piece costs $2.50. What should be the cost of the larger piece?

4. The areas of two similar rectangles are 1875 ft² and 135 ft². Find the ratio of their perimeters.
Lesson 10-5

Trigonometry and Area

1. Finding Area: The radius of a garden is 10 ft. Find the area of the garden.
   - Use the diagram in Example 1. Find the area of the garden.
   - Find the area using the formula: $A = \pi r^2$.
   - Because a pentagon has five sides, $A = \frac{1}{2} b h$.
   - Therefore, $A = \frac{1}{2} \times 65 \times 180$.
   - So $A = 180$.
   - Finally, substitute into the area formula $A = \pi r^2$.

2. Surveying: A triangular park has two sides that measure 200 ft and 300 ft and forms a 60° angle. Find the area of the park to the nearest hundred square feet.
   - Use Theorem 10-8: The area of a triangle is $A = \frac{1}{2} b h$.
   - Use the area formula for a triangle: $A = \frac{1}{2} \times 200 \times 300 \times \sin 60°$.
   - Therefore, $A = \frac{1}{2} \times 200 \times 300 \times \frac{\sqrt{3}}{2}$.
   - Simplify: $A = 17320.5$.
   - The area of the park is approximately $17300$.

Quick Check:

1. Find the area of a regular octagon with a perimeter of 80 ft. Give the area to the nearest tenth.
2. Find the area of a building plot with sides of 120 ft and 90 ft long. They include an angle of 60°. Find the area of the building plot to the nearest square foot.

Local Standards: ____________________________________

Measuring Physical Attributes

Topic:__________________________________________________________

NAEP 2005 Strand: Measurement

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Lesson 10-6 Daily Notetaking Guide

Circles and Area

Lesson Objectives

- Identify the measure of central angles in a circle.
- Find the circumference and area of a circle.

Vocabulary and Key Concepts

- Postulate 10-1: Arc Addition Postulate
- The sum of the measures of two adjacent arcs is the measure of the larger arc.
- Theorem 10-9: Circumference of a Circle
- The circumference of a circle equals $2\pi r$.
- Theorem 10-10: Arc Length
- The length of an arc of a circle is the product of the ratio of the measure of the arc to the circumference of the circle and the circumference of the circle.

A circle is the set of all points equidistant from a given point called the center.

An arc is a segment of a circle.

A central angle is an angle whose vertex is the center of the circle.

A minor arc is an arc whose endpoints are both points on the circle.

A major arc is an arc whose endpoints are both points on the circle.

Quick Check:

1. Use the diagram in Example 1. Find $m\angle XCD$, $m\angle XDF$, $m\angle XFP$, and $m\angle XHP$.

2. Find the area of a semicircle with a radius of 10 cm. Give the area to the nearest tenth.

Local Standards: ____________________________________

Measuring Physical Attributes

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Lesson 10-6 Daily Notetaking Guide

Geometry: All-In-One Answers Version A (continued)
Quick Check.

1. Find the semicircle with radius 1.3 m in terms of 

2. The diameter of a bicycle wheel is 22 in. To the nearest whole number, how many revolutions does the wheel make when the bicycle travels 100 ft?

3. A circle has a diameter of 20 cm. What is the area of a sector bounded by a 20° major arc? Round your answer to the nearest tenth.

Quick Check.

1. The radius of the archery target is ( ft)

2. The radius of the yellow region is about 41 in.²

3. The thickness of fencing material is needed.

Finding the Area of Segments of Circles

A segment of a circle is a region bounded by two radii and their intercepted arc. A segment of a circle is the part bounded by an arc and the segment joining its endpoints.

Example:

Find the area of the segment of the circle.

Step 1: Find the area of sector AOB.

Step 2: Find the area of segment of the circle.

Step 3: Subtract the area of \( \triangle ABC \) from the area of segment of the circle to find the area of the segment of the circle.

Quick Check

1. A circle has a radius of 12 cm. Find the area of the smaller segment of the circle determined by a 60° arc. Round your answer to the nearest tenth.
Lesson 10-8
Geometric Probability

Vocabulary

Geometric probability is a model in which you let points represent outcomes.

Example

1. A point on is selected at random. What is the probability that it is a point on ?

   \[ P(A) = \frac{\text{length of segment } AB}{\text{length of segment } CD} = \frac{8\text{ in.}}{10\text{ in.}} = 0.8 \]

2. The length of the segment between 2 and 10 is 0.02 in. Find the probability that the center of a quarter with a radius of 15 in. must land at least 0.02 in. beyond the boundary of the inner circle in order to lie entirely outside the inner circle. Because the inner circle has a radius of 9 in., the quarter must land entirely within the outer region of the circle at right. Find the area of the region between the square and the circle.

   \[ A = \pi r^2 = \pi (9\text{ in.})^2 = 81\pi \text{ in.}^2 \]

   The probability that the quarter lands entirely within the outer region of the circle is about 0.326 or 32.6%. Use areas to calculate the probability that a dart landing randomly in the square does not land within the circle. The probability that a dart landing randomly in the square does not land within the circle is about %.

   \[ \frac{A_{	ext{outer region}}}{A_{	ext{square}}} = \frac{100\pi \text{ in.}^2}{100\text{ in.}^2} = 0.32573 \]
Lesson 11-2: Surface Areas of Prisms and Cylinders

1. Identify the vertices, edges, and faces of the polyhedron.

There are vertices: V = 6
There are edges: E = 12
There are faces: F = 8

2. Use Euler’s Formula to find the number of edges on a solid with 6 faces and 8 vertices.

Euler’s Formula: V - E + F = 2

Example:
Verify Euler’s Formula for the two-dimensional net of the solid in Example 1.

Solution:

Count the regions: 9
Count the vertices: 9
Count the segments: 12

A solid with 6 faces and 8 vertices has 12 edges.

Quick Check:

a. List the vertices, edges, and faces of the polyhedron.

b. Draw a net for the prism.

c. Verify Euler’s Formula for the two-dimensional net of the prism.

Answers:

A solid with 6 faces and 8 vertices has 12 edges.

---

Lesson 11-2: Surface Areas of Prisms and Cylinders

Vocabulary and Key Concepts:

- **Surface Area of a Prism**: The lateral area of a prism is the product of the perimeter of the base and the height.

- **Surface Area of a Cylinder**: The lateral area of a cylinder is the product of the circumference of the base and the height.

- **Lateral Area of a Prism**: L.A. = Pbh

- **Lateral Area of a Cylinder**: L.A. = \( \pi rh \)

Theorems:

1. **Theorem 11-1**: Lateral and Surface Area of a Prism
   - The lateral area of a right prism is the product of the perimeter of the base and the height.
   - \( \text{L.A.} = P \times h \)

2. **Theorem 11-2**: Lateral and Surface Area of a Cylinder
   - The lateral area of a cylinder is the product of the circumference of the base and the height.
   - \( \text{L.A.} = \pi r \times h \)

Examples:

1. Find the surface area of a cylinder with a radius of 5 cm and a height of 10 cm.

   Surface Area = \( \pi rh \times 2 \)

   \( \text{Surface Area} = \pi \times 5 \times 10 \times 2 = 100\pi \text{ cm}^2 \)

2. Find the surface area of a right prism with a base area of 25 sq cm and a height of 12 cm.

   Surface Area = \( \pi rh + \pi r^2 \times 2 \)

   \( \text{Surface Area} = 25 \times 12 + 25 \times 2 = 350 \text{ sq cm} \)

---

Lesson 11-2: Surface Areas of Prisms and Cylinders

Name _____________________ Class ___________________ Date ______________

---

Example:

Verify Euler’s Formula for the two-dimensional net of the solid in Example 1.

Solution:

Count the regions: 9
Count the vertices: 9
Count the segments: 12

A solid with 6 faces and 8 vertices has 12 edges.

---

Quick Check:

a. Verify Euler’s Formula for the two-dimensional net of the prism.

c. Draw a net for the prism.

Answers:

A solid with 6 faces and 8 vertices has 12 edges.

---

Lesson 11-2: Surface Areas of Prisms and Cylinders

Name _____________________ Class ___________________ Date ______________

---

Example:

Verify Euler’s Formula for the two-dimensional net of the solid in Example 1.

Solution:

Count the regions: 9
Count the vertices: 9
Count the segments: 12

A solid with 6 faces and 8 vertices has 12 edges.

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Quick Check:

a. Verify Euler’s Formula for the two-dimensional net of the prism.

c. Draw a net for the prism.

Answers:

A solid with 6 faces and 8 vertices has 12 edges.

---

Lesson 11-2: Surface Areas of Prisms and Cylinders

Name _____________________ Class ___________________ Date ______________

---

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A solid with 6 faces and 8 vertices has 12 edges.

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Quick Check:

a. Verify Euler’s Formula for the two-dimensional net of the prism.

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A solid with 6 faces and 8 vertices has 12 edges.

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Lesson 11-2: Surface Areas of Prisms and Cylinders

Name _____________________ Class ___________________ Date ______________

---

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Verify Euler’s Formula for the two-dimensional net of the solid in Example 1.

Solution:

Count the regions: 9
Count the vertices: 9
Count the segments: 12

A solid with 6 faces and 8 vertices has 12 edges.

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Quick Check:

a. Verify Euler’s Formula for the two-dimensional net of the prism.

c. Draw a net for the prism.

Answers:

A solid with 6 faces and 8 vertices has 12 edges.

---

Lesson 11-2: Surface Areas of Prisms and Cylinders

Name _____________________ Class ___________________ Date ______________

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Example:

Verify Euler’s Formula for the two-dimensional net of the solid in Example 1.

Solution:

Count the regions: 9
Count the vertices: 9
Count the segments: 12

A solid with 6 faces and 8 vertices has 12 edges.

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Quick Check:

a. Verify Euler’s Formula for the two-dimensional net of the prism.

c. Draw a net for the prism.

Answers:

A solid with 6 faces and 8 vertices has 12 edges.

---

Lesson 11-2: Surface Areas of Prisms and Cylinders

Name _____________________ Class ___________________ Date ______________

---

Example:

Verify Euler’s Formula for the two-dimensional net of the solid in Example 1.

Solution:

Count the regions: 9
Count the vertices: 9
Count the segments: 12

A solid with 6 faces and 8 vertices has 12 edges.

---

Quick Check:

a. Verify Euler’s Formula for the two-dimensional net of the prism.

c. Draw a net for the prism.

Answers:

A solid with 6 faces and 8 vertices has 12 edges.
Lesson 11-3 Surface Areas of Pyramids and Cones

Lesson Objectives

✓ Find the surface area of a pyramid
✓ Find the surface area of a cone

Vocabulary and Key Concepts

Theorem 11-3: Lateral and Surface Area of a Regular Pyramid

The lateral area of a regular pyramid is half the product of the perimeter of the base and the slant height.

\[ L.A. = \frac{1}{2} \times p \times s \]

The surface area of a regular pyramid is the sum of the lateral area and the area of the base.

\[ S.A. = L.A. + B \]

A regular pyramid is a pyramid whose base is a regular polygon and whose lateral faces are congruent triangles.

The altitude of a pyramid or a cone is the perpendicular segment from the vertex to the plane of the base.

The height of a pyramid or a cone is the length of the altitude.

The slant height of a regular pyramid is the length of the lateral face.

The lateral area of a pyramid is the sum of the areas of the congruent lateral faces.

The surface area of a pyramid is the sum of the lateral area and the area of the base.

Examples

1. Finding Surface Area of a Pyramid

Find the surface area of a square pyramid with base edges 7.5 ft and slant height 12 ft.

The perimeter of the base is: \[ p = 4 \times 7.5 = 30 \text{ ft} \]

The slant height is: \[ s = 12 \text{ ft} \]

The lateral area is: \[ L.A. = \frac{1}{2} \times p \times s = \frac{1}{2} \times 30 \times 12 = 180 \text{ ft}^2 \]

The area of the base is: \[ B = 7.5 \times 7.5 = 56.25 \text{ ft}^2 \]

The surface area is: \[ S.A. = L.A. + B = 180 + 56.25 = 236.25 \text{ ft}^2 \]
Lesson 11-4 Volumes of Prisms and Cylinders

**Lesson Objectives**
- Find the volume of a prism
- Find the volume of a cylinder

**Vocabulary and Key Concepts**
- **Theorem 11-5: Cavalieri's Principle**
  - If two space figures have the same height and the same cross sectional area at every level, then they have the same volume.

- **Theorem 11-6: Volume of a Prism**
  - The volume of a prism is the product of the area of the base and the height of the prism.
  - \( V = Bh \)

- **Theorem 11-7: Volume of a Cylinder**
  - The volume of a cylinder is the product of the area of the base and the height of the cylinder.
  - \( V = 

**Examples**

1. **Finding Volume of a Cylinder**
   - Find the volume of the cylinder.
   - Leave your answer in terms of \( \pi \).
   - The formula for the volume of a cylinder is \( V = \pi r^2 h \).
   - The diagram shows \( r \) and \( h \), but you must find \( r \).
   - \( V = \pi r^2 h \)
   - Substitute.
   - \( V = \pi \times 3^2 \times 16 \)
   - Simplify.
   - The volume of the cylinder is \( 144\pi \) cubic inches.

2. **Finding Volume of a Triangular Prism**
   - Find the volume of the triangular prism.
   - The prism is a right triangular prism with triangular base.
   - The base of the triangular prism is a right triangle where one leg is \( 8 \) in. and the other leg is \( 6 \) in.
   - The area of the base is \( \frac{1}{2} \times 8 \times 6 = 24 \) in.\(^2\).
   - Use the formula for the volume of a prism.
   - \( V = Bh \)
   - Substitute.
   - \( V = \times 24 \times 14 \)
   - Simplify.
   - The volume of the triangular prism is \( 672 \) cubic inches.

3. **Finding Volume of a Composite Figure**
   - Find the volume of the composite space figure.
   - You can use three rectangular prisms to find the volume.
   - Each prism's volume can be found using the formula \( V = Bh \).
   - Volume of Prism I = \( 8 \times 6 \times 4 = 192 \) cubic inches.
   - Volume of Prism II = \( 8 \times 6 \times 4 = 192 \) cubic inches.
   - Volume of Prism III = \( 8 \times 6 \times 4 = 192 \) cubic inches.
   - Sum of the volumes = \( 576 \) cubic inches.
   - The volume of the composite space figure is \( 576 \) cubic inches.

**Quick Check**

1. **Volume of the triangular prism.**
   - \( 120 \text{ ft}^3 \)

2. **The cylinder shown is oblique.**
   - a. Find its volume in terms of \( \pi \).
   - \( 250 \text{ ft}^3 \pi \)
   - b. Find its volume to the nearest tenth of a cubic meter.
   - \( 400.2 \text{ m}^3 \)
   - c. Find the volume of the composite space figure.
   - \( 32 \text{ in}^3 \)
Lesson 11-5
Volumes of Pyramids and Cones

Example 1 Finding Volume of a Pyramid Find the volume of a square pyramid with base edges 15 cm and height 22 cm.

Because the base is a square, the area of the base is

\[ B = 15 \times 15 = 225 \text{ cm}^2 \]

Use the formula for the volume of a pyramid.

\[ V = \frac{1}{3} Bh \]

\[ V = \frac{1}{3} \times 225 \times 22 = 1616 \text{ cm}^3 \]

The volume of the square pyramid is 1616 cm³.

Example 2 Finding Volume of a Cone Find the volume of the cone

\[ V = \frac{1}{3} Bh \]

\[ V = \frac{1}{3} \times \pi \times 12 \times 17 = 704 \text{ m}^3 \]

The volume of the cone is 704 m³.

Lesson Objectives
- Find the volume of a pyramid.
- Find the volume of a cone.

NAEP 2005 Strand: Measurement
Topic: Measuring Physical Attributes

Local Standards: ____________________________

Topic: Measuring Physical Attributes

Step 1 Find the height of the pyramid.

\[ 5^2 = 9^2 + h^2 \]

\[ h = \sqrt{9^2 - 5^2} = \sqrt{34} \]

Step 2 Find the square root of each side.

\[ h \approx 5.8 \text{ in.} \]

The volume of the square pyramid is

\[ V = \frac{1}{3} \times 16 \times 5.8 \approx 128 \text{ in.}^3 \]

The volume of the cone is

\[ V = \frac{1}{3} \times \pi \times 12 \times 10 \approx 125.7 \text{ in.}^3 \]

Local Standards: ____________________________
Lesson 11-7 Areas and Volumes of Similar Solids

### Vocabulary and Key Concepts

- **Similar solids** have the same shape and all of their corresponding parts are proportional.

- The similarity ratio of two similar solids is the ratio of their corresponding linear dimensions.

### Examples

#### Identifying Similar Solids

- Find the similarity ratio of two similar cylinders with surface areas of 90π ft² and 24π ft².

  - Use the ratio of the surface areas to find the similarity ratio.

  \[
  \frac{24\pi}{90\pi} = \frac{4}{15}
  \]

  - Simplify.

  \[
  \frac{4}{15}
  \]

  - Take the square root of each side.

  \[
  \frac{2}{\sqrt{15}}
  \]

  - The similarity ratio is \( \frac{2}{\sqrt{15}} \).

#### Using a Similarity Ratio

- Two similar square pyramids have volumes of 45 cm³ and 162 cm³. The surface area of the larger pyramid is 135 cm². Find the surface area of the smaller pyramid.

  - Step 1: Find the similarity ratio.

    \[
    \frac{135}{45} = 3
    \]

  - The ratio of the volumes is \( \frac{3}{1} \).

  - Take the cube root of each side.

    \[
    \sqrt[3]{3} = 1.442
    \]

  - The similarity ratio is \( \sqrt[3]{3} \).

  - Step 2: Use the similarity ratio to find the surface area \( S_2 \) of the smaller pyramid.

    \[
    \frac{S_2}{S_1} = (\sqrt[3]{3})^2
    \]

    \[
    S_2 = \left(\frac{S_1}{(\sqrt[3]{3})^2}\right)
    \]

    \[
    S_2 = \frac{45}{3} = 15
    \]

  - The surface area of the smaller pyramid is 15 cm².
Quick Check

1. Are the two cylinders similar? If so, give the similarity ratio.

2. Find the similarity ratio of two similar prism with surface areas 144 m² and 324 m².

3. The volumes of two similar solids are 128 m³ and 250 m³. The surface area of the larger solid is 250 m². What is the surface area of the smaller solid?

Lesson 12-1

Tangent Lines

Lesson Objectives

• Use the relationship between a radius and a tangent
• Use the relationship between two tangents from one point

Vocabulary and Key Concepts

Theorem 12-1

If a line is tangent to a circle, then the line is perpendicular to the radius drawn at the point of tangency.

Theorem 12-2

If a line in the plane of a circle is perpendicular to a radius at its endpoint on the circle, then the line is tangent to the circle.

Theorem 12-3

The two segments tangent to a circle from a point outside the circle are congruent.

Examples

1. Finding a Tangent

\( \triangle O \) has radius 5. Point \( P \) is outside \( \triangle O \) such that \( PO = 12 \), and point \( A \) is in \( \triangle O \) such that \( PA = 13 \) is tangent to \( \triangle O \) at \( A \). Explain.

2. Circles Inscribed in Polygons

\( \triangle C \) is inscribed in quadrilateral \( XYZW \). Find the perimeter of \( XYZW \).

Examples

1. Finding a Tangent

\( \triangle O \) has radius 5. Point \( P \) is outside \( \triangle O \) such that \( PO = 12 \), and point \( A \) is in \( \triangle O \) such that \( PA = 13 \) is tangent to \( \triangle O \) at \( A \). Explain.

2. Circles Inscribed in Polygons

\( \triangle C \) is inscribed in quadrilateral \( XYZW \). Find the perimeter of \( XYZW \).
Lesson 12-2 Chords and Arcs

**Lesson Objectives**
- Identify congruent chords, arcs, and central angles.
- Use properties of lines through the center of a circle.
- Solve problems involving chords, arcs, and central angles.

**Vocabulary and Key Concepts**

1. Congruent chords have congruent arcs.
2. Congruent arcs have congruent central angles.
3. Congruent central angles have congruent chords.
4. A chord is a segment whose endpoints are on a circle.
5. A circle is the set of all points in a plane that are equidistant from a given point, the center.

**Quick Check**

1. If \( \overline{BC} \) is a chord of \( \odot O \) and \( \overline{BC} \) is congruent to \( \overline{AB} \), what can you conclude about the circle?

2. Find the value of \( x \) in the circle at the right.

3. Find the length of the chord to the nearest unit.

4. Find the distance from the midpoint of the chord to the midpoint of its perpendicular bisector.

5. Use the Pythagorean Theorem.

6. Use the circle at the right.

7. Use congruent chords, arcs, and central angles.

**Examples**

**Example 1**
In the diagram, \( \overline{BC} \) is a chord of \( \odot O \).

- Using Theorem 12-1, \( \overline{BC} \) is congruent to \( \overline{AB} \).

- Using Theorem 12-2, \( \overline{BC} \) is congruent to \( \overline{AB} \).

- Using Theorem 12-3, \( \overline{BC} \) is congruent to \( \overline{AB} \).

**Example 2**
In the diagram, \( \odot O \) is tangent to \( \odot A \) at \( A \).

- Using Theorem 12-4, \( \overline{BC} \) is congruent to \( \overline{AB} \).

- Using Theorem 12-5, \( \overline{BC} \) is congruent to \( \overline{AB} \).

- Using Theorem 12-6, \( \overline{BC} \) is congruent to \( \overline{AB} \).
Lesson 12-3

Inscribed Angles

Vocabulary and Key Concepts

Theorem 12-9: Inscribed Angle Theorem
The measure of an inscribed angle is
\[ \frac{1}{2} \text{ the measure of the intercepted arc}. \]

Theorem 12-10
The measure of an angle formed by a tangent and a chord is
\[ \frac{1}{2} \text{ the measure of the intercepted arc}. \]

Corollaries to the Inscribed Angle Theorem
1. Two inscribed angles that intercept the same arc are congruent.
2. An angle inscribed in a semicircle is a right angle.
3. The opposite angles of a quadrilateral inscribed in a circle are supplementary.

Quick Check
1. For the diagram at the right, find the measure of each numbered angle.

Example
Using the Inscribed Angle Theorem
1. Find the values of \( x \) and \( y \).

Local Standards: ____________________________________

Lesson Objectives
- The measure of an angle formed by a tangent and a chord is half the measure of the intercepted arc.
- The opposite angles of a quadrilateral inscribed in a circle are supplementary.
- The measure of an angle formed by two lines that constant along any line through the point and circle.
- The sum of the measures of the three angles of the triangle inscribed in \( O \) is 180°. Therefore, the angle whose intercepted arc has measure 3 must have measure 180° – 58° = 122°.
- Because the angle inscribed in a circle and sides that are chords of the circle.

Local Standards: ____________________________________

Lesson 12-4

Angle Measures and Segment Lengths

Vocabulary and Key Concepts

Theorem 12-11
The measure of an angle formed by two lines that intersect inside a circle is half the sum of the measures of the intercepted arcs.

Theorem 12-12
For a given point and circle, the product of the lengths of the two segments from the point to the circle is constant along any line through the point and circle.
Lesson 12-5

Lesson Objectives
- Find the center and radius of a circle

Key Concepts

Theorem 12-13
The standard form of an equation of a circle with center \((h, k)\) and radius \(r\)

\[(x - h)^2 + (y - k)^2 = r^2\]

Examples

1. Writing the Equation of a Circle

Write the standard equation of a circle with center \((3, 4)\) and radius 5.

\[(x - 3)^2 + (y - 4)^2 = 5^2\]

2. Using the Center and a Point on a Circle

Write the standard equation of a circle with center \((2, -3)\) that passes through the point \((-1, 5)\).

First find the radius.

\[r = \sqrt{(x - h)^2 + (y - k)^2}\]

\[r = \sqrt{(-1 - 2)^2 + (5 - (-3))^2}\]

\[r = \sqrt{9 + 64}\]

\[r = \sqrt{73}\]

Then find the standard equation of the circle with center \((2, -3)\) and radius \(\sqrt{73}\).

\[(x - 2)^2 + (y + 3)^2 = 73\]

Graphing a Circle Given Its Equation

Find the center and radius of the circle with equation \(x^2 + y^2 - 6x + 8y + 9 = 0\). Write the equation in the standard form.

To find the center, you need to complete the square for both \(x\) and \(y\).

\[(x^2 - 6x) + (y^2 + 8y) = -9\]

Add \(9\) and \(16\) to both sides.

\[(x^2 - 6x + 9) + (y^2 + 8y + 16) = -9 + 9 + 16\]

\[(x - 3)^2 + (y + 4)^2 = 16\]

The center is \((3, -4)\) and the radius is \(4\).

Quick Check

1. Write the standard equation of each circle.

a. center \((3, 0)\); radius 2

\[x^2 + y^2 - 6x = 0\]

b. center \((-2, 0)\); radius \(\sqrt{2}\)

\[x^2 + y^2 + 4x = 2\]
2. Write the standard equation of the circle with center \((2, 5)\) that passes through the point \((-1, 3)\).

\[ (x - 2)^2 + (y - 5)^2 = 13 \]

3. Find the center and radius of the circle with equation \((x - 3)^2 + (y - 2)^2 = 100\). Then graph the circle.

Center: \((3, 2)\); radius: 10

4. When you make a call on a cellular phone, a tower receives the call. In the diagram, the set of points in a plane that are 6 cm from point \(P\) is a circle with center \(P\) and radius 6 cm. Describe the set of points in a plane that are 6 cm from point \(Q\).

The set of points in a plane that are 6 cm from point \(P\) is a circle with center \(P\) and radius 6 cm.

The set of points in a plane that are 6 cm from point \(Q\) is a circle with center \(Q\) and radius 6 cm. Because the sum (12 cm) of the radius is greater than 6 cm, the circles overlap.

Draw a segment \(PQ\) to show the two points where the circles intersect.

Describe the locus of points in a plane that are 2 cm from line \(XY\).

The locus of points in a plane that are 2 cm from line \(XY\) is a circle with center \(A\) and radius 2 cm.

Use a compass to draw a circle with center \(B\) and radius 2 cm. The circles intersect at two points, so the locus is the two points where the circles intersect.

Describe the locus of points in space that are 4 cm from a plane \(M\).

Imagine plane \(M\) as a horizontal plane. Because distance is measured along a perpendicular segment from a point to a plane, the locus of points in space that are 4 cm from a plane \(M\) is a plane \(X\) above and parallel to plane \(M\) and another plane \(Y\) below and parallel to plane \(M\).